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# cONSTRICTION/SPREADING RESISTANCE MODEL FOR ELECTRONICS PACKAGING

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## ABSTRACT

An analytical model is developed for predicting constriction and spreading resistances associated with heat transfer from various electronic components under different modes of cooling. The model assumes a heat source in contact with a larger cold plate which is in turn cooled with a convective heat transfer coefficient specified over the sink surface. Unlike existing models that mostly assume limiting boundary conditions, such as isothermal conditions, at the sink surface, the present solution allows one to accurately determine the constriction and spreading resistances in a plate with a full range of cooling conditions varying from an isothermal condition in one limit to a uniform heat-flux condition in the other limit. Dimensionless expressions in the form of infinite series are provided for computing the average and maximum constriction resistances as a function of relative contact size, plate thickness and the Biot number. The results are compared with published numerical data, and the agreement is excellent over a wide range of parameters typically found in microelectronics applications.

## NOMENCLATURE

| | |
|---|---|
| $A_s$ | source area, $m^2$ |
| $A_p$ | plate area, $m^2$ |
| $a$ | source radius, $m$ |
| $Bi$ | Biot number $\equiv hb/k$ |
| $b$ | plate radius, $m$ |
| $h$ | heat transfer coefficient, $W/m^2K$ |
| $J_i(\cdot)$ | Bessel function of the first kind of order $i$ |
| $k$ | thermal conductivity, $W/mK$ |
| $Q$ | rate of heat transfer, $W$ |
| $q$ | heat flux at the source surface, $W/m^2$ |
| $R_c$ | constriction resistance, $^0C/W$ |
| $R_f$ | external resistance, $^0C/W$ |
| $R_m$ | material resistance, $^0C/W$ |
| $r, z$ | cylindrical coordinates, $m$ |
| $T$ | temperature excess over ambient temperature, $^0C$ |
| $T_s$ | temperature at the source, $^0C$ |
| $t$ | plate thickness, $m$ |

### Greek Symbols

| | |
|---|---|
| $\gamma, \zeta$ | dimensionless coordinates, $r/b$, $z/b$ |
| $\epsilon$ | dimensionless contact radius, $a/b$ |
| $\lambda_c$ | empirical parameter given by Eq. (27) |
| $\lambda_n$ | eigenvalue |
| $\tau$ | dimensionless plate thickness, $t/b$ |
| $\Phi_c$ | dimensionless parameter defined by Eq. (26) |
| $\Phi_n$ | dimensionless parameter defined by Eq. (21) |
| $\Psi$ | dimensionless constriction resistance |

### Subscripts

| | |
|---|---|
| $ave$ | average |
| $max$ | maximum |

## INTRODUCTION

Constriction and spreading resistances exist whenever heat flows from one region to another of different cross sectional area. The term constriction is used to describe the situation where heat flows into a narrower region, and

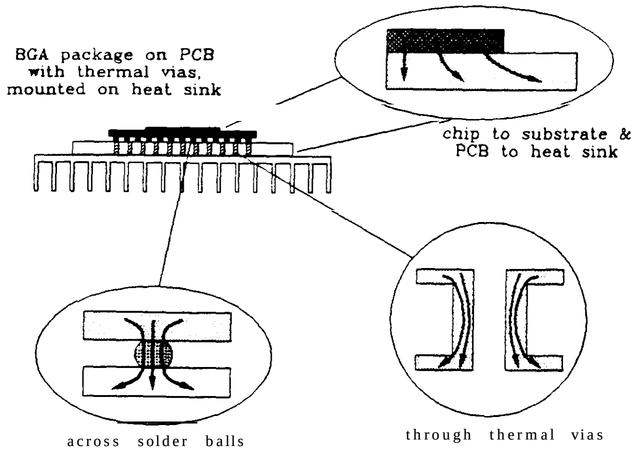


Figure 1: Constriction and Spreading Resistances in Electronics Packaging

spreading is used to describe the case where heat flows out of a narrow region into a larger cross sectional area. Except for the purpose of examining the effects of thermal rectification due to the direction of heat flow, the constriction and spreading resistances are often assumed to be identical in magnitude in a given system and, therefore, may be used interchangeably. Figure 1 depicts a typical example where combinations of such resistances occur at different locations within a path of heat dissipation. In many instances, the constriction and spreading resistances may become greater than one-dimensional material resist antes in a system, and represent an essential part of the total thermal resistance in predicting the overall thermal performance of the device.

Many studies have been carried out to determine the constriction/spreading resistances in a system similar to the one investigated herein. Nelson and **Sayers (1992)** used a control volume based finite difference method to compute two-dimensional planar and axi-symmetric spreading resistances with a uniform heat source and a uniform external thermal resistance at the sink surface. Dimensionless resistances are tabulated for ranges of relative source size and the inverse of the external resistance, defined as the Biot number. Kennedy (1960) obtained analytical solutions for axi-symmetric problems with a uniform heat-flux source on a finite cylinder, but only considered isothermal boundary conditions over the sink surface. Design charts are provided over a range of geometric parameters for cases where isothermal boundary conditions are specified over different surfaces. However, the assumption of isothermal surfaces at the cooling side substantially limits the applicability of the solutions in many practical problems and, in some cases, results in constriction resistance values that are underpredicted by as much as orders of magnitude. A general solution for the thermal constriction resistance due to a flux applied over a circular portion of the upper surface of a two-layer compound disk with a film coefficient over the lower surface has been presented by Yovanovich et al. (1979). Negus and Yovanovich (1987) developed a thermal modeling procedure for predicting the temperature distribution of a semiconductor die with multiple heat sources. They considered general thermal boundary conditions over the top surface of the die. However, as in Kennedy's solution, isothermal conditions are assumed along the bottom of the die. Mikic (1966), Cooper et al. (1969), and Yovanovich and Schneider (1977) presented resistances for problems involving semi-infinite domains, such as a uniform heat-flux source on a half-space or a semi-infinite heat-flux tube.

Although there exists a large number of solutions and calculated data available for obtaining various types of constriction and spreading resistances, a simple, practical solution that is capable of dealing with general boundary conditions over the sink surface has not been developed. With this in mind, the present investigation was carried out to obtain analytical solutions to a problem that involves general boundary conditions. The resulting solutions, described in this paper, are of the infinite series form, and are an extension to Kennedy's isothermal solution (1960). Then, baaed on the present solutions, a set of closed form, algebraic approximations is developed by the current authors for calculating the average and maximum constriction resistances. The final approximate expressions are included herein, but details of the development are presented elsewhere (Song et al., 1994).

Naraghi and Antonetti (1993) used a numerical method to compute constriction resistances of **a** single heat source of various shapes located on a heat-flux tube of different contours. As noted by Yovanovich and his coworkers in their studies dealing with a contact source on a half-space (Yovanovich, 1976; Yovanovich and Burde, 1977; Yovanovich et al., 1977), they demonstrated that the dimensionless constriction resistances are a weak function of the shape of the contact configuration when the square root of the contact area is used as the characteristic length and the area ratio is kept constant: see Figure 2. The relative difference is shown to become greater as the area ratio increases. However, at large area ratios, the constriction resist ante is often much smaller than other resistances in the system that the impact or the difference introduced in the constriction resistance of the total system resistance is usually insignificant. The finding allows the constriction and spreading resistance obtained for one contact shape to be readily used

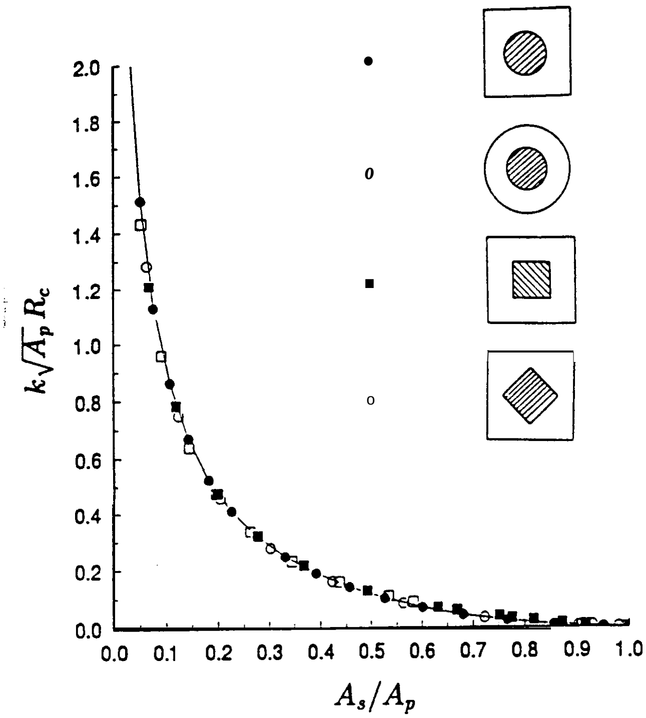


Figure 2: Dimensionless Resistance versus Dimensionless Source Area for Different Source Contours and Flux-Tubes (Naraghi and Antonetti, 1993)

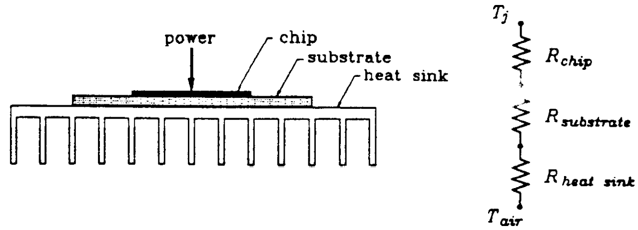


Figure 3: Example Problem and Thermal Network

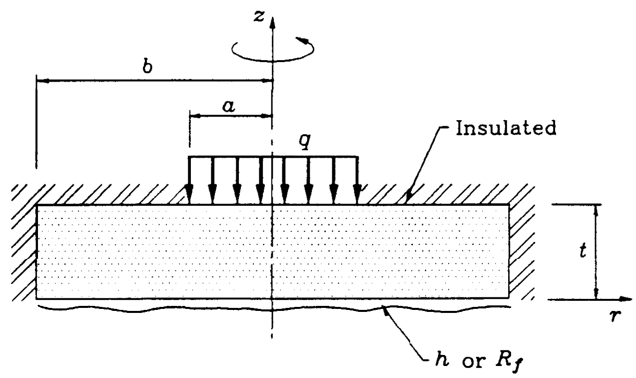


Figure 4: Thermal Modeling

other configurations without introducing a large discrepancy. The same exercise has been practiced in other studies with great success (Yovanovich, 1992; Lee et al., 1992, 1993).

## THERMAL MODELING

Consider an example problem shown in Figure 3. A device is mounted on a substrate which is attached to a heat sink with a known average external sink-to-ambient thermal resistance. Since the majority of the heat dissipated by the device will flow through the substrate and into the heat sink, the heat loss through the other exposed surfaces may be assumed negligible, and the thermal system can be approximated by a simple serial resist ante network included in the figure. The thermal resistances shown in the figure represent the total resistances of the corresponding parts. For the chip and substrate, they can be considered as a combination of one-dimensional material and spreading resistances and, in the case of the heat sink, the total resistance consists of the sink-to-ambient resistance and the spreading resistance in the base plate.

Many semiconductor devices are square or rectangular. However, as discussed in the previous section, the spreading resistances associated with the different parts of the system shown in Figure 3 can be determined by considering the axi-symmetric problem described in Figure 4, provided that the square root of the area is used as the characteristic length, and the same area ratio is used. Heat enters the top surface of the plate over a concentric circular surface of radius $a$ and leaves the plate of radius $b$ through the bottom surface over which a uniform heat transfer coefficient, or an external resistance is prescribed. The remaining top and side surfaces are assumed to be adiabatic. For non-circular devices, the equivalent contact and plate radii are obtained as follows:

$$a = \sqrt{\frac{A_s}{\pi}} \qquad (1)$$

$$b = \sqrt{\frac{A_p}{\pi}} \qquad (2)$$

where $A_s$ is the contact area of the heat source, and $A_p$ is the area of the base plate.

The governing differential equation for this problem is Laplace's equation in a two-dimensional cylindrical coordinate system:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2} = 0 \qquad (3)$$

The boundary conditions are

$$\text{at } r=0, \qquad \frac{\partial T}{\partial r}=0 \tag{4}$$

$$\text{at } r=b, \qquad \frac{\partial T}{\partial r}=0 \tag{5}$$

$$\text{at } z=0, \qquad k\frac{\partial T}{\partial z}=hT \tag{6}$$

$$\text{at } z=t, \qquad k\frac{\partial T}{\partial z}=\begin{cases} q & \text{for } 0<r\leq a \\ 0 & \text{for } a<r\leq b \end{cases} \tag{7}$$

where $k$ is the thermal conductivity, and $T$ is the local temperature excess over the ambient fluid temperature. The heat source is represented by $q$, denoting the uniform heat flux over the contact area, and $h$ is the constant heat transfer coefficient at the bottom surface. If an external thermal resistance, $R_f$, is specified as the boundary condition for the bottom surface, the following expression can be used for conversion:

$$h=\frac{1}{R_f A_p} \tag{8}$$

Upon using the method of separation of variables, the solution for the temperature distribution in the plate is obtained as

$$T = \frac{qa}{k}\left[\epsilon\left(\frac{1}{Bi}+\zeta\right) + 2\sum_{n=1}^{\infty}\frac{J_1(\lambda_n\epsilon)J_0(\lambda_n\gamma)}{\lambda_n^2 J_0^2(\lambda_n)}\frac{cosh(\lambda_n\zeta)}{cosh(\lambda_n\tau)}\frac{\tanh(\lambda_n\zeta)+\frac{\lambda_n}{Bi}}{1+\frac{\lambda_n}{Bi}\tanh(\lambda_n\tau)}\right] \tag{9}$$

where $\epsilon = a/b$, $\tau = t/b$, $\zeta = z/b$ and $\gamma = r/b$. $J_0(\cdot)$ and $J_1(\cdot)$ are the Bessel functions of the first kind of order O and 1, and the eigenvalue $\lambda_n$ is the $n$-th root of the transcendental equation which satisfies the adiabatic boundary condition at $\gamma = 1$:

$$J_1(\lambda_n)=0 \tag{10}$$

Also, the Biot number is defined as

$$Bi=\frac{hb}{k} \tag{11}$$

The average and maximum total resist antes are defined as

$$R_{ave} \equiv \frac{T_{s_{ave}}}{Q} = R_f+R_m+R_{c_{ave}} \tag{12}$$

$$R_{max} \equiv \frac{T_{s_{max}}}{Q} = R_f+R_m+R_{c_{max}} \tag{13}$$

where the area-averaged temperature over the contact region and the maximum local temperature at the center of the contact can be determined from the following expressions using the temperature solution given by Eq. (9):

$$T_{s_{ave}} = \frac{1}{A_s}\int_0^{A_s} T(\zeta=\tau)\,dA \tag{14}$$

$$T_{s_{max}} = T(\gamma=0,\zeta=\tau) \tag{15}$$

and Q is the total rate of heat flow:

$$\mathbf{Q} = qA \tag{16}$$

In Equations (12) and (13), $R_f$ and $R_m$ denote the external and material resistances*, given as

$$R_f = \frac{1}{hA_p} \tag{17}$$

$$R_m = \frac{t}{kA_p} \tag{18}$$

and $R_{c_{ave}}$ and $R_{c_{max}}$ are the average and maximum constriction resistances. Upon evaluating Equations (14) and (15), closed-form expressions for the dimensionless constriction resistances can be obtained by rearranging Equations (12) and (13) as

$$\Psi_{ave} \equiv k\sqrt{A_s}\,R_{c_{ave}} = \frac{4}{\sqrt{\pi}\,\epsilon}\sum_{n=1}^{\infty}\frac{J_1^2(\lambda_n\epsilon)}{\lambda_n^3 J_0^2(\lambda_n)}\Phi_n \tag{19}$$

$$\Psi_{max} \equiv k\sqrt{A_s}\,R_{c_{max}} = \frac{2}{\sqrt{\pi}}\sum_{n=1}^{\infty}\frac{J_1(\lambda_n\epsilon)}{\lambda_n^2 J_0^2(\lambda_n)}\Phi_n \tag{20}$$

where

$$\Phi_n = \frac{\tanh(\lambda_n\tau)+\frac{\lambda_n}{Bi}}{1+\frac{\lambda_n}{Bi}\tanh(\lambda_n\tau)} \tag{21}$$

## RESULTS AND DISCUSSION

Table 1 contains a list of solutions available for the dimensionless average constriction resistances. The superscript $T$ indicates that the results are for the cases with an isothermal source rather than with a uniform flux one. Also, in order to account for the differences in the choice of the characteristic length, the values of $\Psi^T$ shown in the table are modified from the reported values by a factor of $\sqrt{\pi}/2$. The generality of the present solution can be easily seen from the table as it becomes identical to the other analytical solutions under limiting conditions: the present solution becomes Kennedy's isothermal solutions as $Bi$ approaches infinity and further becomes identical to Yovanovich's infinite-t solution as $\tau$ becomes large.

Table 1: Available Solutions and Correlations for $\Psi_{ave}$

| Reference | Boundary Conditions | Solution |
|---|---|---|
| Mikic, 1966 | $b \to \infty$ | $\Psi^T_{ave} = \frac{\sqrt{\pi}}{4}$ |
| Yovanovich and Schneider, 1977 | $t \to \infty$ | $\Psi_{ave} = 0.479$ |
| Cooper et al. 1969 | $t \to \infty$ | $\Psi^T_{ave}(\epsilon) = \frac{\sqrt{\pi}}{4}(1 - \epsilon)^{3/2}$ |
| Yovanovich 1992 | | $\Psi_{ave}(\epsilon) = \frac{4}{\sqrt{\pi}\,\epsilon} \sum_{n=1}^{\infty} \frac{J_1^2(\lambda_n \epsilon)}{\lambda_n^3 J_0^2(\lambda_n)}$ |
| Kennedy, 1960 | *iso-T* at $z = 0$ | $\Psi_{ave}(\epsilon, \tau) = \frac{4}{\sqrt{\pi}\,\epsilon} \sum_{n=1}^{\infty} \frac{J_1^2(\lambda_n \epsilon) \tanh(\lambda_n \tau)}{\lambda_n^3 J_0^2(\lambda_n)}$ |
| Present | hor Rfatz=O | $\Psi_{ave}(\epsilon, \tau, Bi) = \frac{4}{\sqrt{\pi}\,\epsilon} \sum_{m=1}^{\infty} \frac{J_1^2(\lambda_n \epsilon)}{\lambda_n^3 J_0^2(\lambda_n)} \frac{\tanh(\lambda_n \tau) + \frac{\lambda_n}{Bi}}{1 + \frac{\lambda_n}{Bi} \tanh(\lambda_n \tau)}$ |

The dimensionless constriction resistant= obtained in the previous section have been computed. Polynomial approximations (Abramowitz and Stegun, 1970) are used in evaluating the Bessel functions, and the Newton-Raphson method is used in determining the eigenvalues. The computation typically requires no more than 100 terrns of the series solutions to obtain resist ante values with a 4 decimal place accuracy, each caae taking less than a second using a personal computer.

The results are compared with the numerical data of Nelson and Sayers (1992) in Figures 5 through 9 for $\epsilon$ values ranging from 0.05 to 0.833, and $Bi$ from O to infinity. As can be seen from the figures, near perfect agreement is shown between the present solution and the numerical results over the full range of parameters examined herein. As the plate thickness increases beyond $\tau \approx 0.6$, it is observed that the constriction resistance becomes independent of both the plate thickness and the

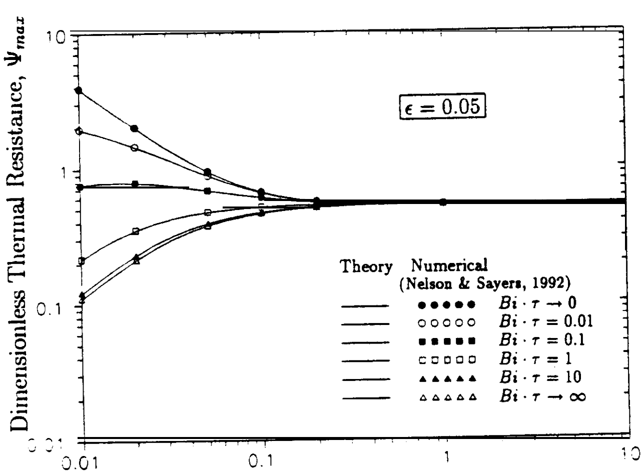


Figure 5: Comparison of Theory with Numerical Solution for Dimensionless Source Area $\epsilon = 0.05$

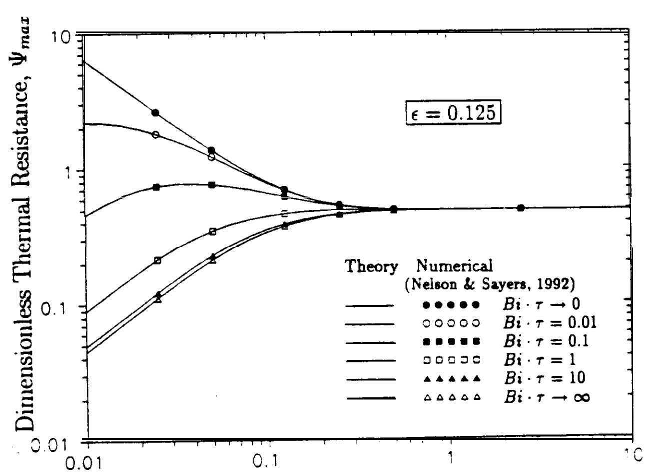


Figure 6: Comparison of Theory with Numerical Solution for Dimensionless Source Area $\epsilon = 0.125$

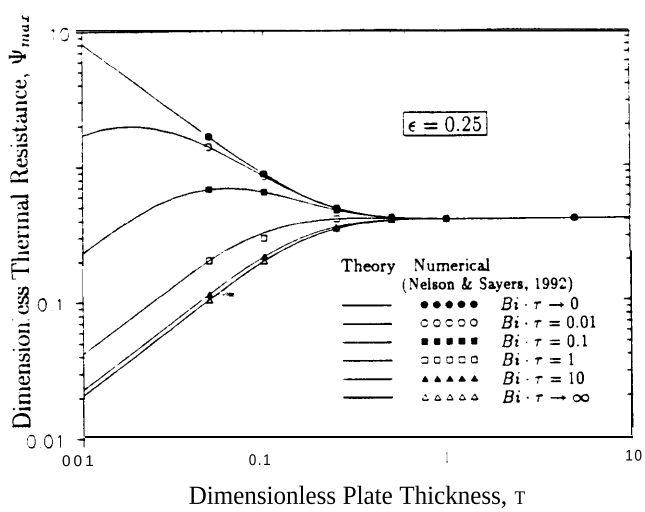


Figure 7: Comparison of Theory with Numerical Solution for Dimensionless Source Area $\epsilon$ = 0.25

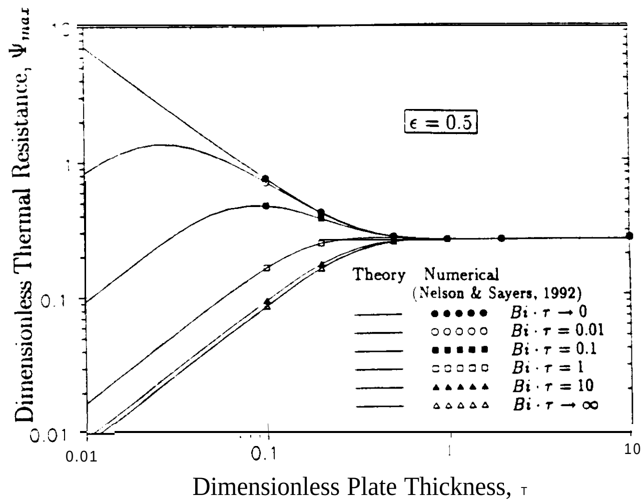


Figure 8: Comparison of Theory with Numerical Solution for Dimensionless Source Area $\epsilon$ = O.5

Biot number, leaving the relative contact size $\epsilon$ as the only parameter upon which the constriction resistance primarily depends. This singular dependency on $\epsilon$ is in accordance with the infinite-t solutions shown in Table 1, and was expected since the pattern of heat flow near the source region would not be affected by the changes in conditions far away from the source. This also illustrates that, as observed by Mikic (1966), the constriction is a rather localized phenomenon confined mostly in the vicinity of a heat source.

For fixed $\epsilon$ and $\tau$, the maximum constriction resistance is observed when the Biot number approaches O. In this limit, the bottom surface becomes iso-flux, and the resistance value obtained under this condition represents the upper bound for the resistance of an actual situation where the Biot number would always be finite and nonzero, The minimum constriction resistance, or the lower bound, corresponds to the case where the Biot number approaches infinity. In this limit, the bottom surface becomes isothermal.

Comparison with Kennedy's isothermal solution (1960) is provided in Figure 10 for $\epsilon$ = 0.1. Note the large discrepancy that may have been introduced if Kennedy's solution was used for cases where the plate is relatively thin and the Biot number, the dimensionless heat transfer coefficient, is small. For example, consider a case

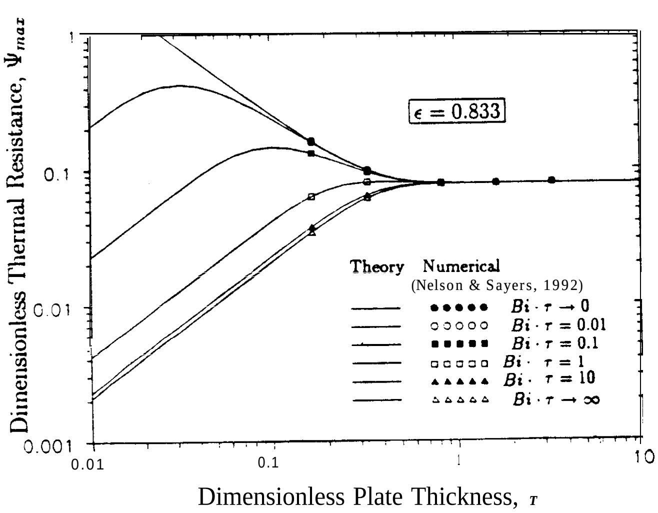


Figure 9: Comparison of Theory with Numerical Solution for Dimensionless Source Area $\epsilon$ = 0.833

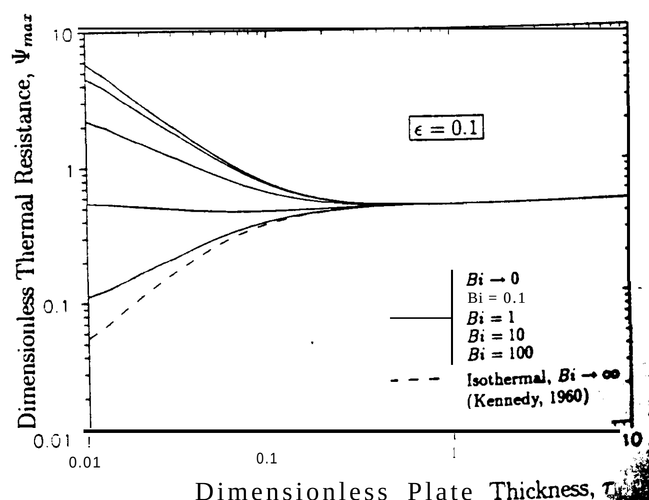


Figure 10: Comparison of Present Solution with Kennedy's Isothermal Solution for Dimensionless Source Area $\epsilon$ = 0.1

where $\epsilon = \tau = 0.1$, and $Bi = 1.0$. For this case, the dimensionless constriction resistance is 0.641, and the non-dimensionalized material and external film resistances, obtained from

$$k\sqrt{A_s}\,R_m = \frac{\epsilon\,\tau}{\sqrt{\pi}} \quad (1)$$

$$k\sqrt{A_s}\,R_f = \frac{\epsilon}{\sqrt{\pi}\,Bi} \quad (2)$$

are 0.006 and 0.056, respectively, resulting in the total value of 0.703. On the other hand, if the isothermal solution was used, it would have yielded 0.376 for the constriction resistance and 0.438 for the total resistance, resulting in an underprediction of the actual total resistance by 38%.

Based on the solutions presented in this paper, simple approximations are developed by the present authors for the dimensionless constriction resistances (Song et al., 1994). They are rewritten here as

$$\Psi_{ave} = \frac{1}{2}(1-\epsilon)^{3/2}\,\Phi_c \quad (3)$$

$$\Psi_{max} = \frac{1}{\sqrt{\pi}}(1-\epsilon)\,\Phi_c \quad (4)$$

where

$$\Phi_c = \frac{\tanh(\lambda_c\tau) + \frac{\lambda_c}{Bi}}{1 + \frac{\lambda_c}{Bi}\tanh(\lambda_c\tau)} \quad (5)$$

with

$$\lambda_c = \pi + \frac{1}{\sqrt{\pi}\,\epsilon} \quad (6)$$

As reported by Song et al., the above correlations agree with the present analytical solutions well within 10% over the range of parameters commonly found in microelectronics applications.

## CONCLUSIONS

The constriction and spreading resistances in a plate with a uniform heat-flux region on one surface and a thermal boundary condition of the third kind prescribed over the other surface have been investigated analytically. Closed form expressions are obtained for the dimensionless average and maximum constriction resistances as functions of three independent variables, namely relative contact radius, plate thickness and the Biot number. The results are presented over a wide range of parameters typically found in a variety of microelectronics applications. Comparisons with existing numerical data resulted in near perfect agreement. It was found, for relatively thick plates ($\tau \geq 0.6$), that the constriction resistance is insensitive to the changes in both the plate thickness and the Biot number, and becomes solely depend on the relative contact size of the heat source. Kennedy's isothermal solution represents the lower bound for the constriction resistance, and it severely underestimates actual resistance values if the plate is relatively thin and the Biot number is not large. Additionally, simple correlations are developed as an extension to the present study and are included here for practical estimation of the resistances.

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