Reducing Noise by Repetition: An Introduction to Signal Averaging

HunerKada Presentation @ Experimental Physics Laboratory

Umer Hassan

SSE, LUMS

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What’s the need?

Small Scale signals are usually swamped by undesired noise. This is evident in many areas,

- Nanotechnology, e.g. measuring resistance of a gold Nano wire.
- NMR spectroscopy when the spectral peaks from carbon-13 nuclei are too small.
- Biomedical or Physiological signals e.g. ECG, Pulse Oximetry, EEG are embedded with noise.

Solution to Problem

- Acquire signal for longer duration or repetitions
- Perform Base line correction i.e. removing offset voltages.
- Finally, averaging the repetitions

An Averaging Process

Consider v(k) is the contaminated signal. Mathematically,

$v(k) = vs(k) + vnoise(k)$,

where

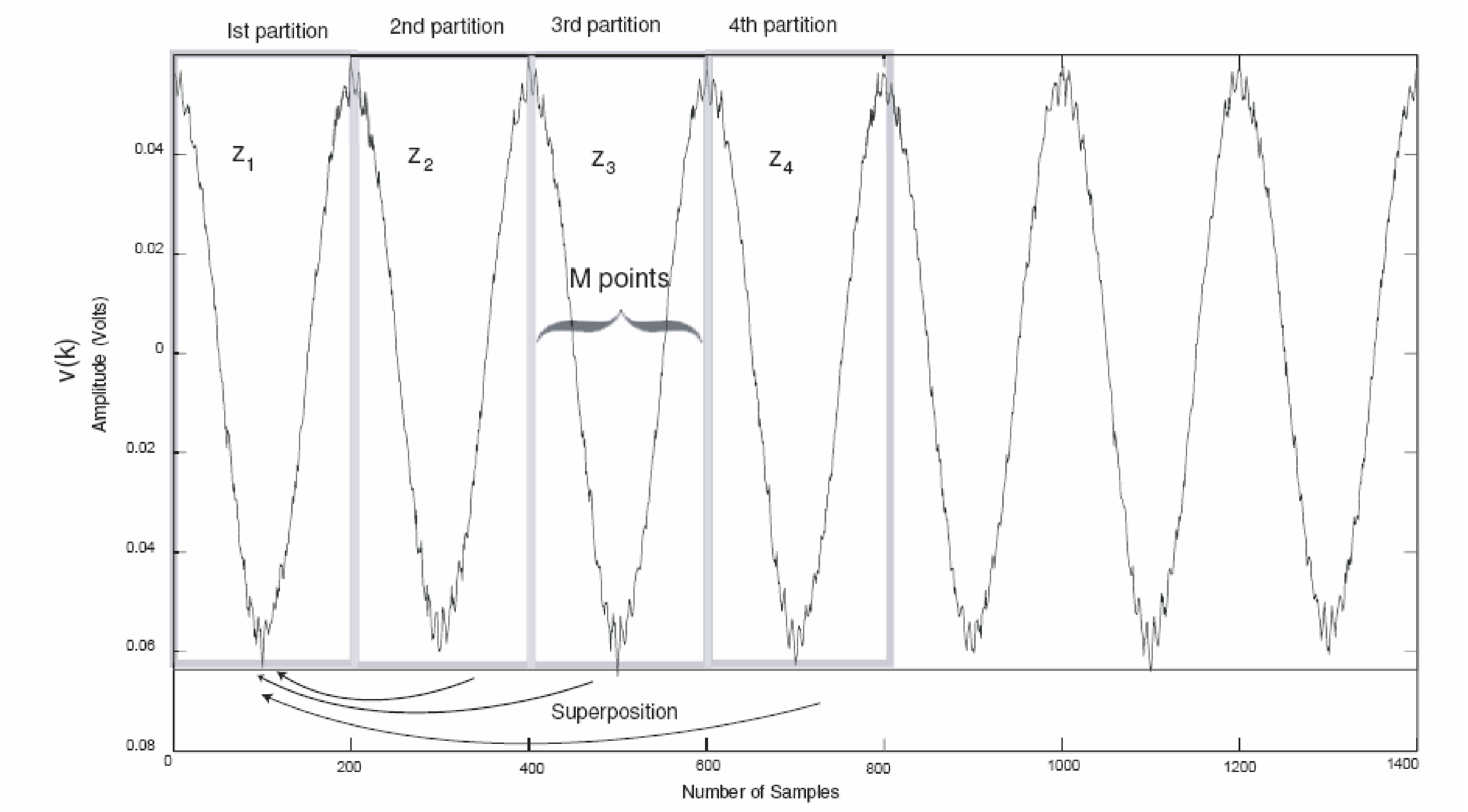
$vs(k)$ being the desired periodic signal

$vnoise(k)$ the unwanted noise

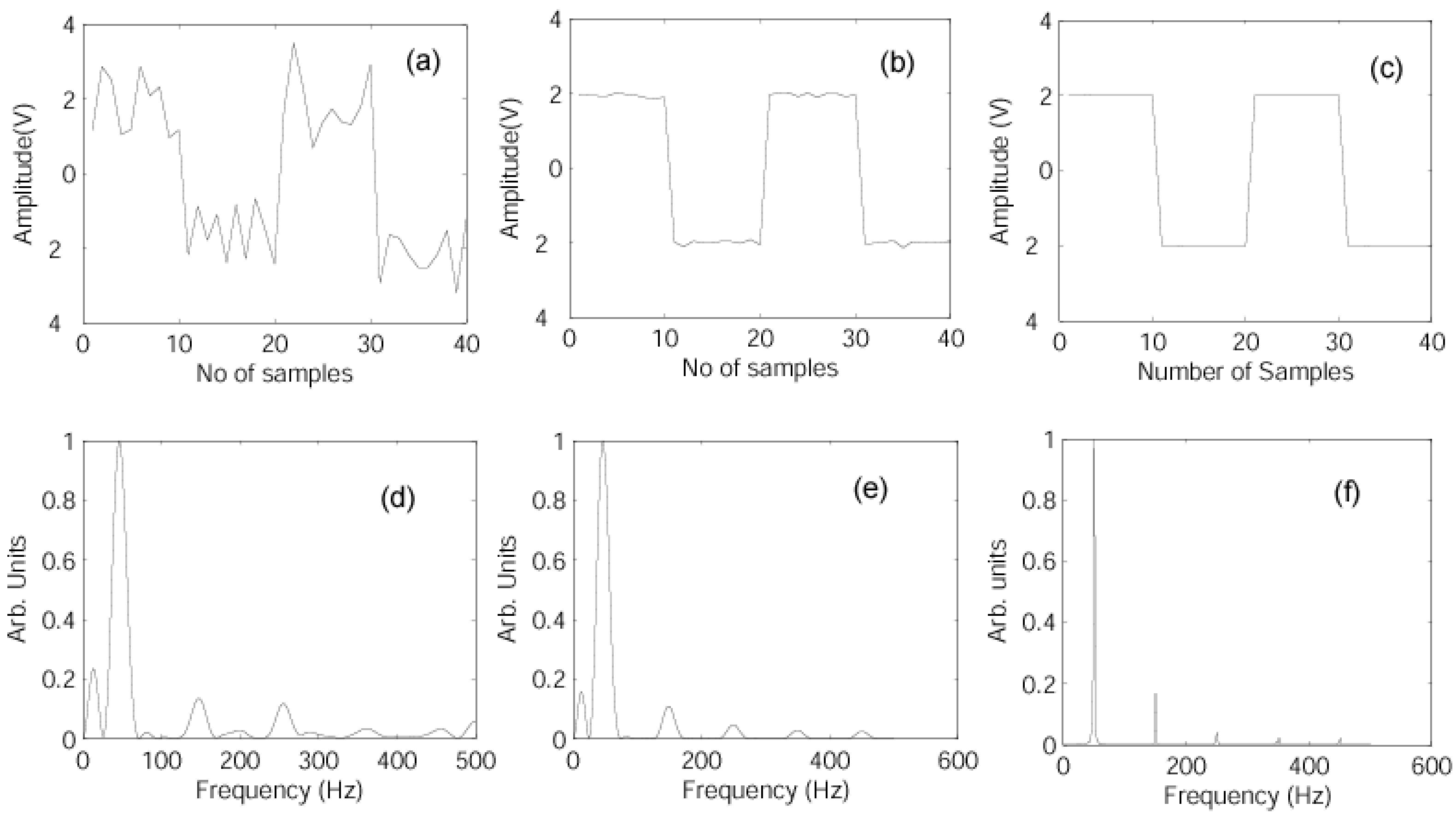
- *Recipe*:

Signal averaging is performed by accumulating and partitioning $v(k)$, and adding the partitions with the hope that the noise adds destructively while the desired signal builds up.

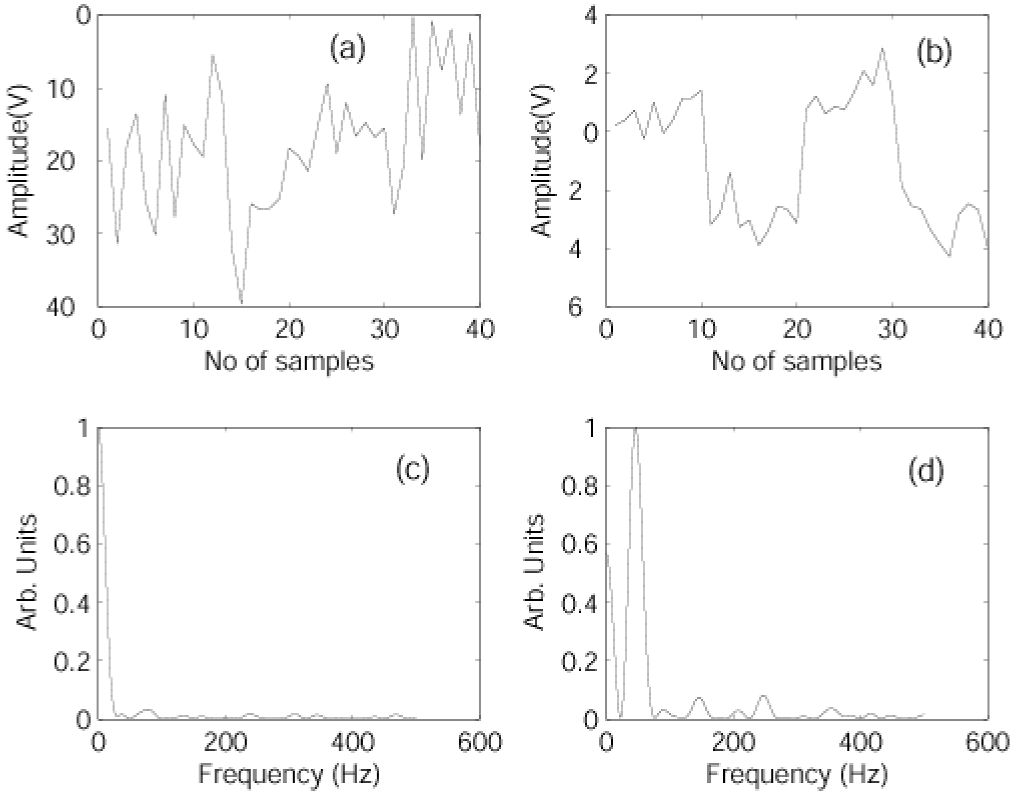
Averaging Explained….



Averaging of Noisy Square Wave for 400 scans



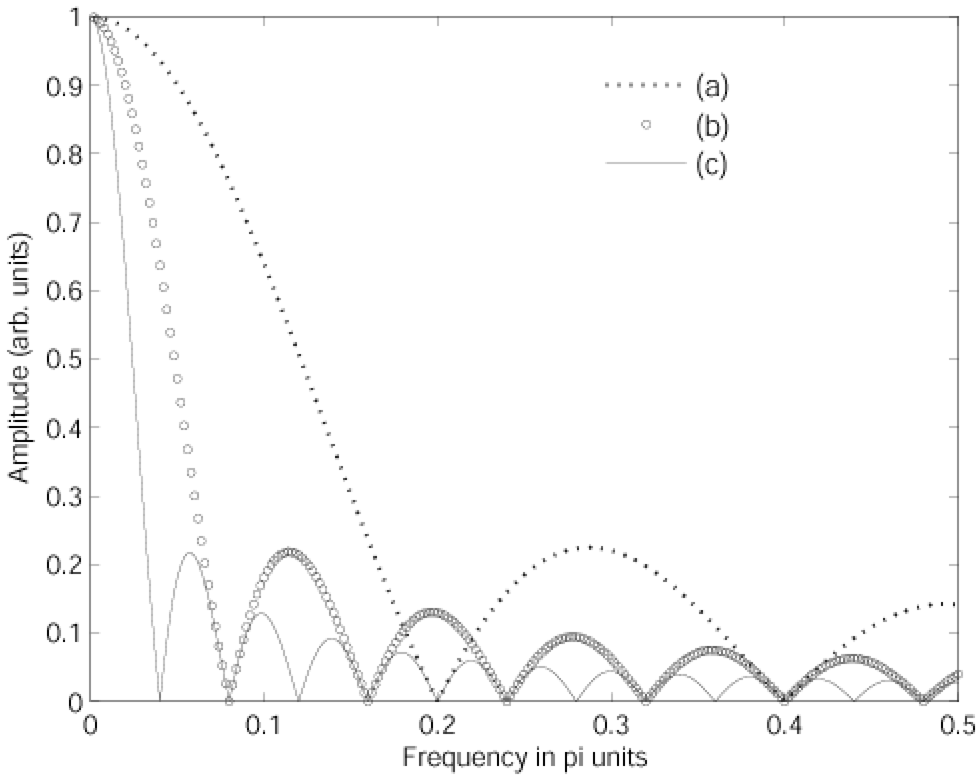
Averaging of pink noise



averaging for 400 scans

Averaging as a digital filter

For 10, 25 and 50 Scans



Mathematical description…

- N are total number of scans, M total points in a single partition

$$y(k) = \frac{\Sigma_{i=1}^{N} z_i(k)}{N}.$$

$$h(k) = \frac{\Sigma_{i=1}^{N} \delta_i(k)}{N}.$$

$$h(k) = \frac{\delta(0) + \delta(M) + \delta(2M) + \ldots + \delta(M(N-1))}{N}.$$

$$\begin{aligned} H(e^{i\omega}) &= \frac{1 + e^{-i\omega M} + e^{-i\omega 2M} + \ldots + e^{-i\omega M(N-1)}}{N} \\ &= \frac{1}{N}\left(\frac{1 - e^{-i\omega MN}}{1 - e^{-i\omega}}\right). \end{aligned}$$

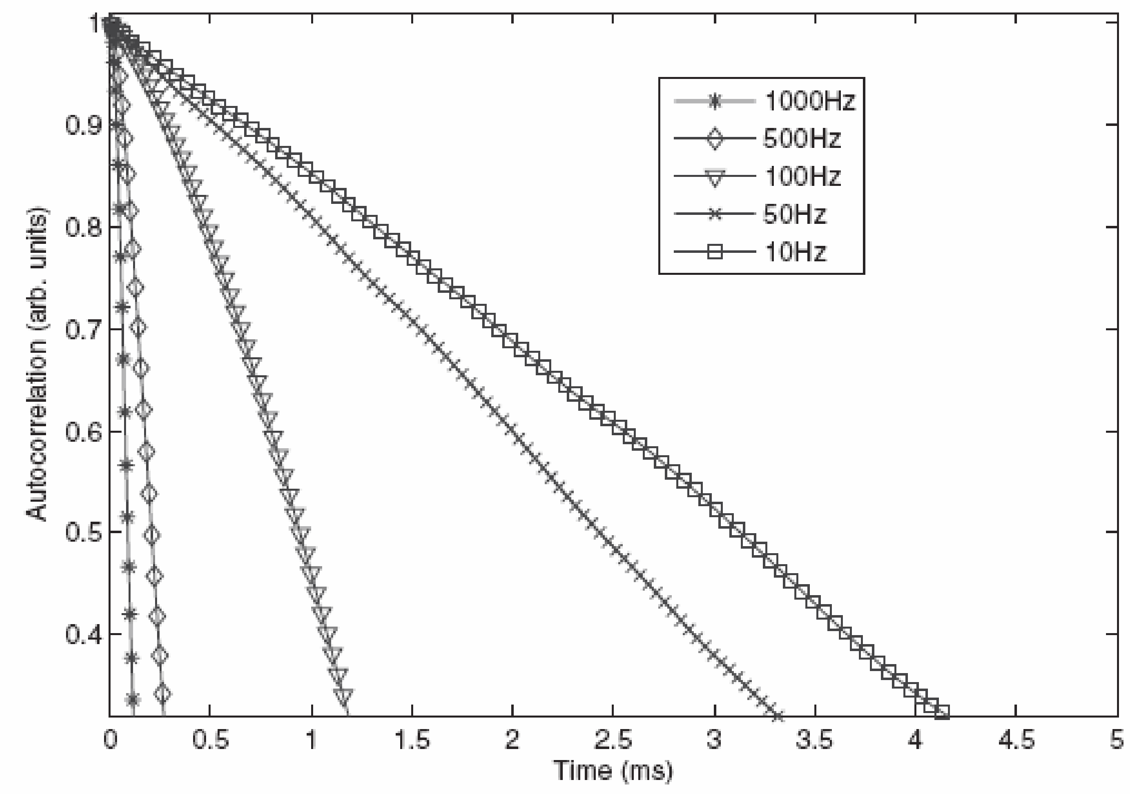
Effect of averaging on the autocorrelation

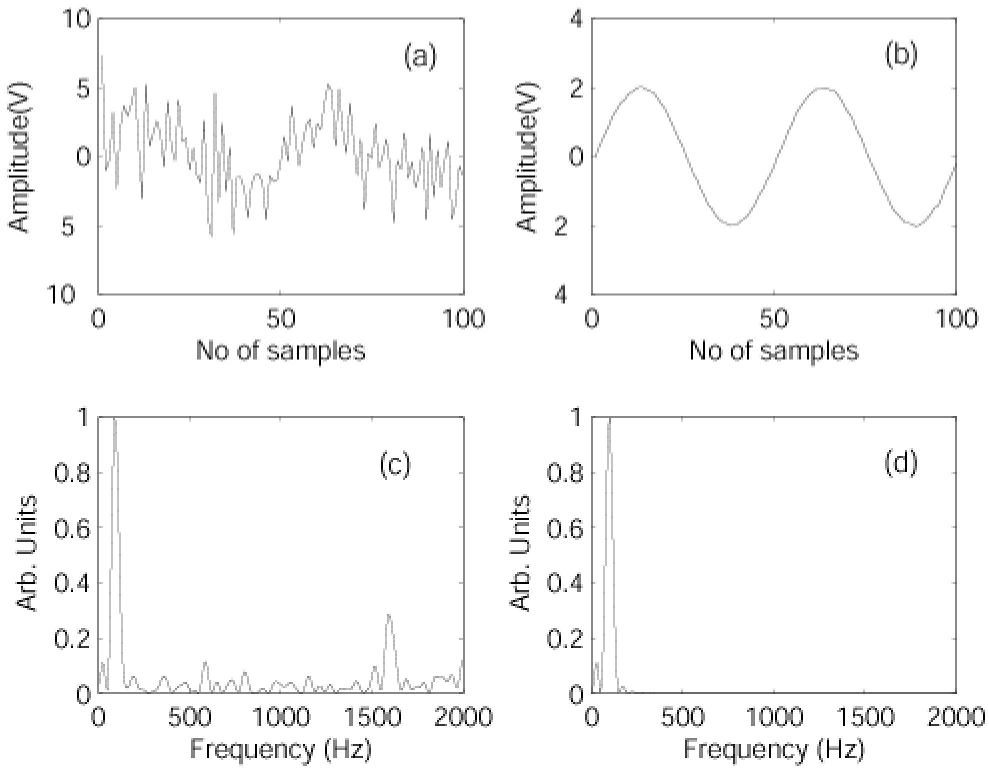
$$G(l) = \frac{1}{M}\Sigma_{k=1}^{M} v(k)v(l+k)$$
$$= \frac{1}{M}\Sigma_{k=1}^{M} (v_s(k) + v_n(k))(v_s(l+k) + v_n(l+k)).$$

$\Sigma_{k=1}^{M} v_n(k)v_s(l+k)$ and $\Sigma_{k=1}^{M} v_s(k)v_n(l+k)$ will be zero

$$G(l) = \frac{1}{M}\left[\Sigma_{k=1}^{M} v_n(k)v_n(l+k) + \Sigma_{k=1}^{M} v_s(k)v_s(l+k)\right].$$

Normalized autocorrelation plot for different cut-off frequencies of Gaussian

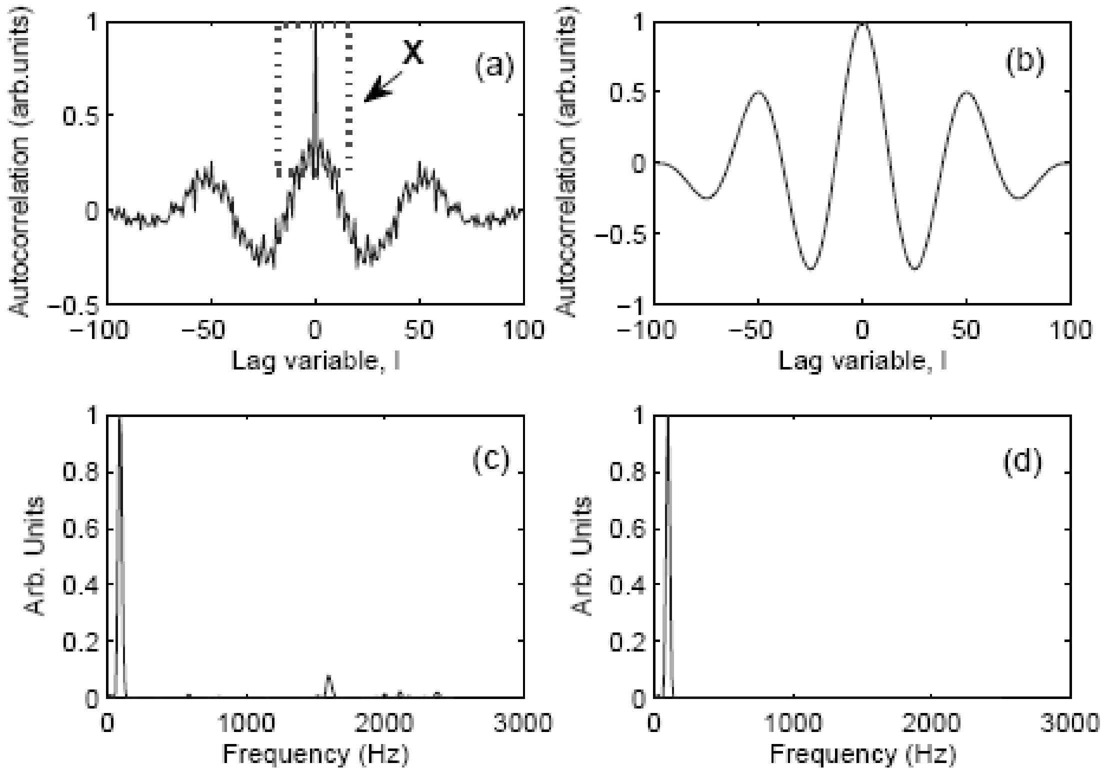




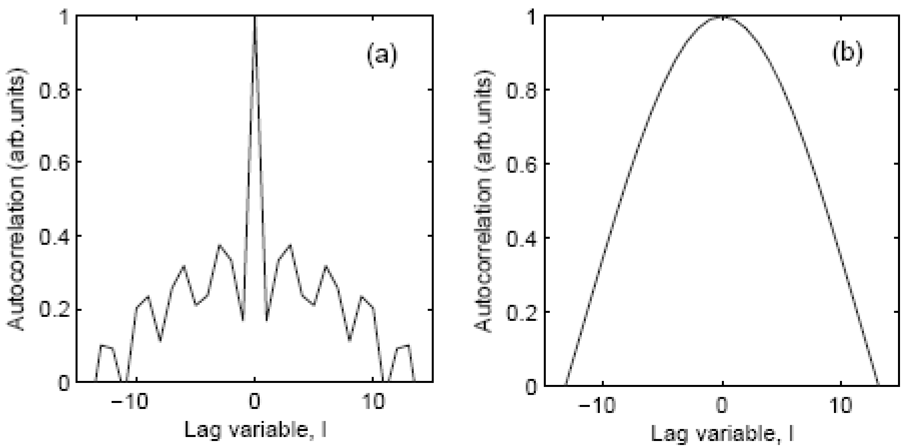
frequency 100 Hz

averaging with 10000 repetitions.

Effect of Averaging on autocorrelation…

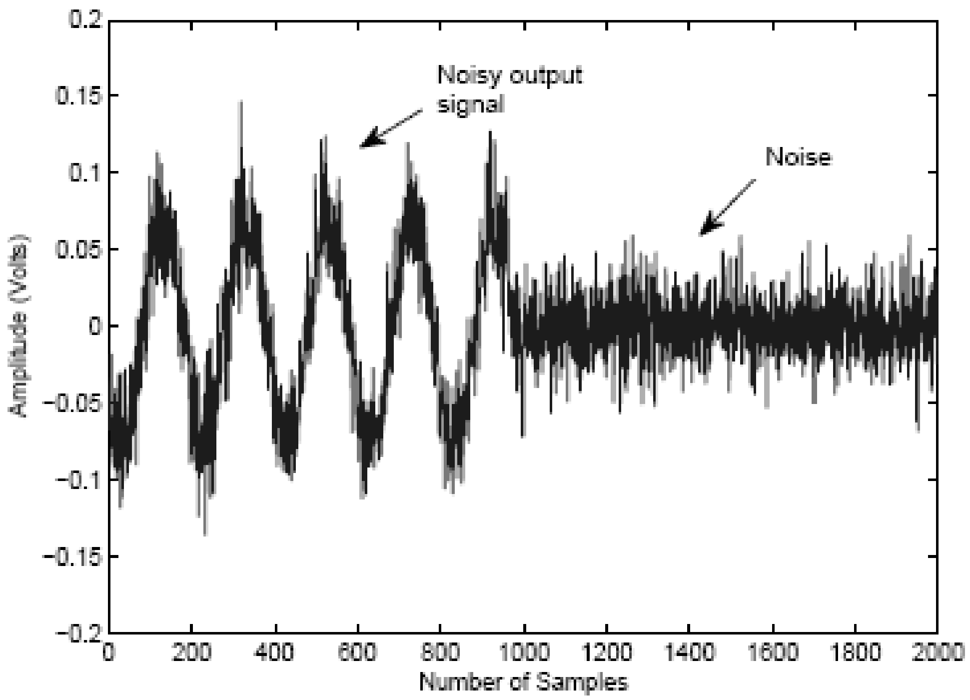


Continued…

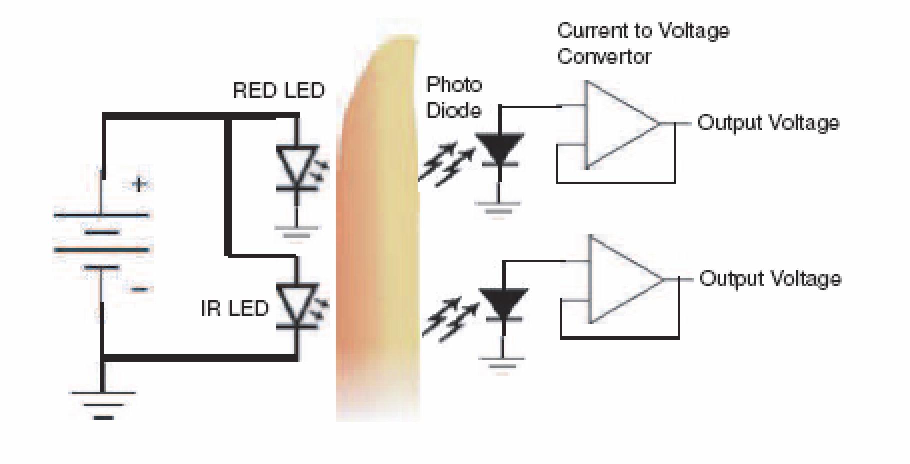


(a)
Autocorrelation (arb.units)
Lag variable, l
1
0.8
0.6
0.4
0.2
0
−10
0
10
(b)
Autocorrelation (arb.units)
Lag variable, l
1
0.8
0.6
0.4
0.2
0
−10
0
10

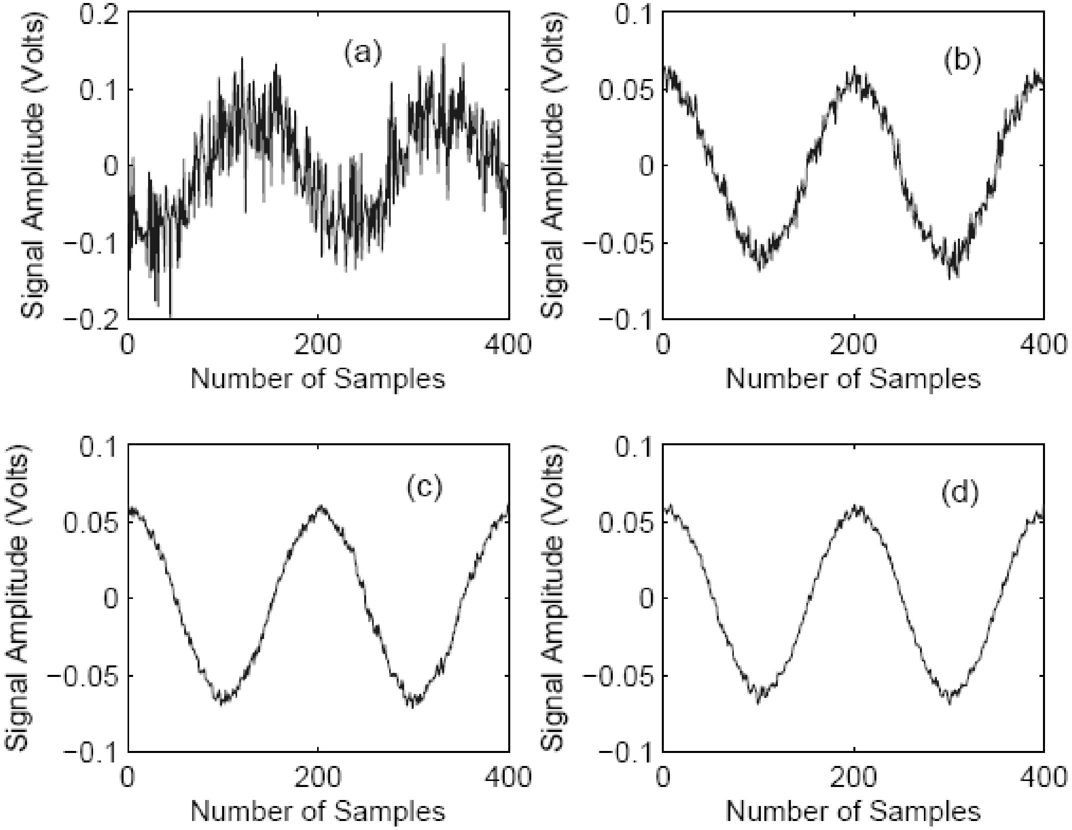
Identifying Signal and Noise



Experimental Setup for Pulse Oximetry



Averaging Results

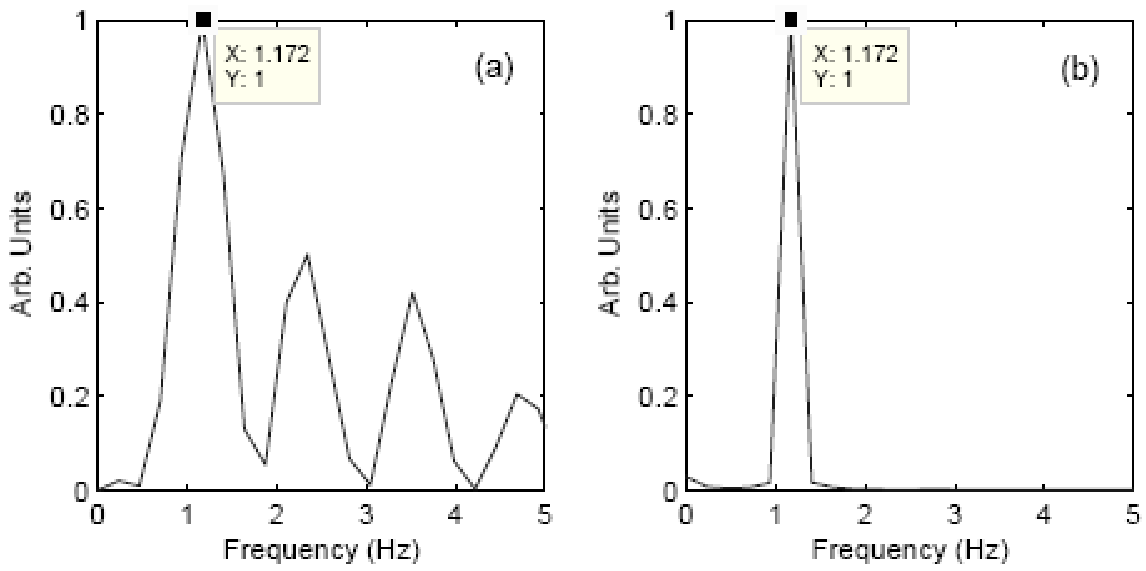


(b) 50 repetitions

(c) 250 repetitions

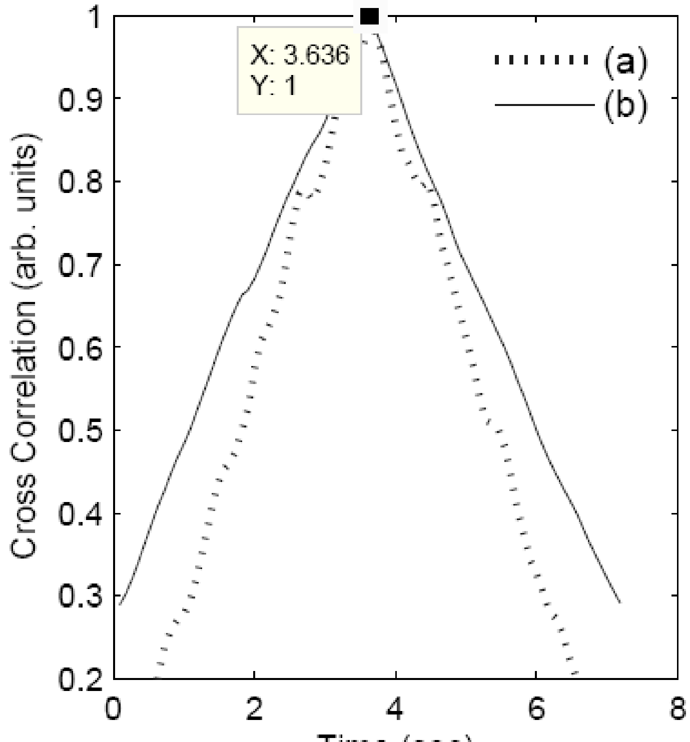
(d) 500 repetitions.

Effect of averaging on the frequency spectrum

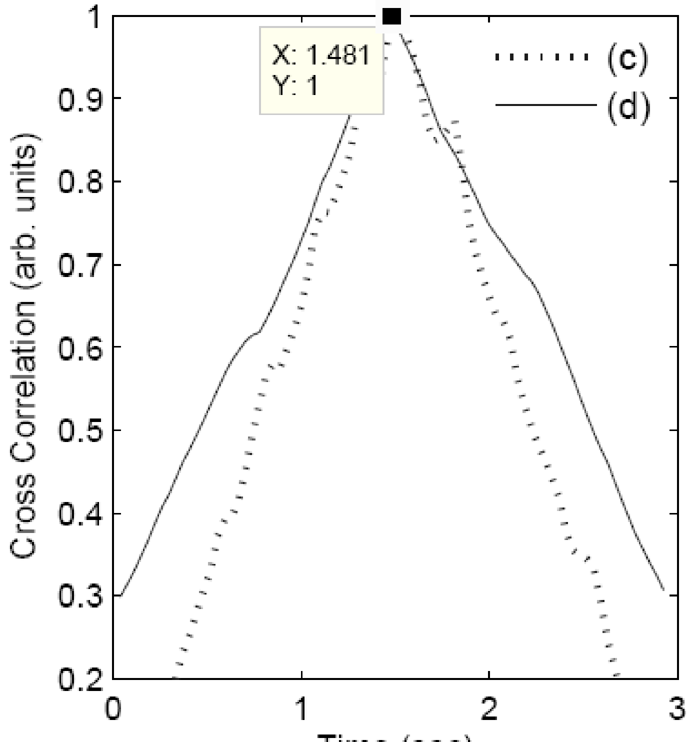


Using correlation data for estimating blood flow velocity

$$S(l) = (1/M)\Sigma_{n=0}^{M-1} f(n)\ g(l+n)$$

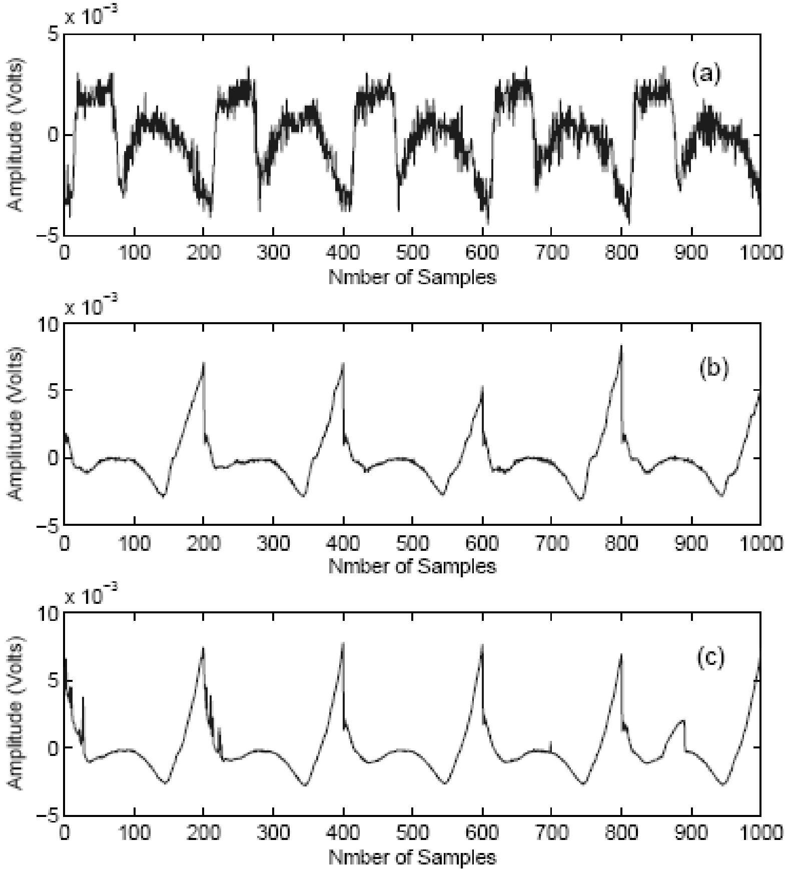


before running



after running

Averaging ECG Signal



50 repetitions

500 repetitions

Reference:

- Umer Hassan, Muhammad Sabieh Anwar, *“Reducing Noise by Repetition: Introduction to Signal Averaging” submitted to* ***Eur. J. Phys****, Dec. 2009.*

Thanks