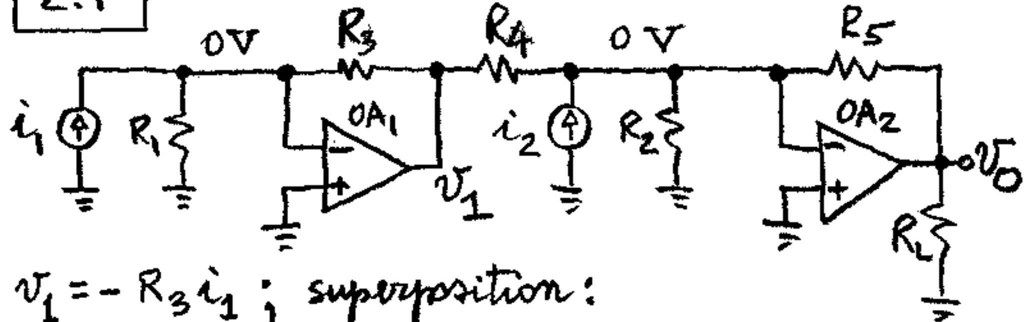
2.1

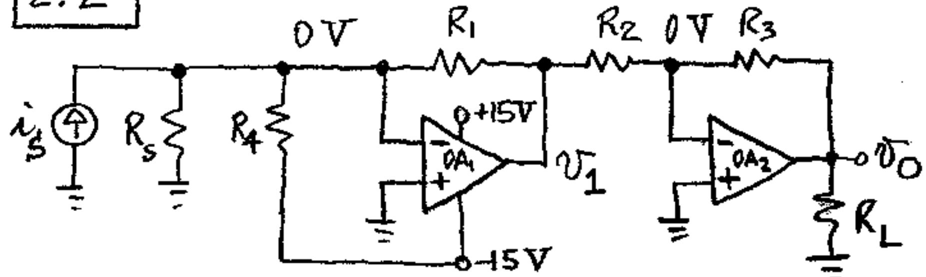


$v_1 = -R_3 i_1$; superposition:

$v_O = -R_5 i_2 - (R_5/R_4)v_1 = R_5\left(\frac{R_3}{R_4}v_1 - v_2\right)$. Use

$R_3 = R_4 = R_5 = 0.1/10^{-6} = 100\ \text{k}\Omega$.

2.2



$v_O = -\frac{R_3}{R_2}v_1 = -\frac{R_3}{R_2}\left[-R_1 i_S - \frac{R_1}{R_4}(-15\ \text{V})\right]$.

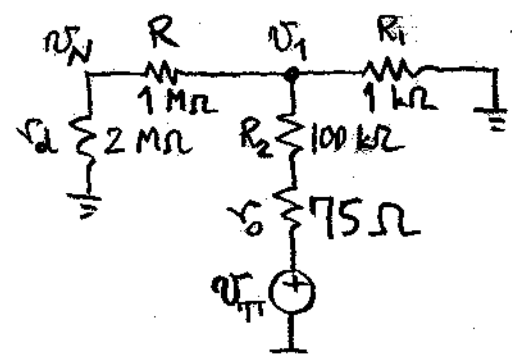
$i_S = 4\ \text{mA} \Rightarrow v_O = 0 \Rightarrow \frac{15}{R_4} = 4 \Rightarrow R_4 = 3.75\ \text{k}\Omega$

(Use 3.74 kΩ, 1%). For simplicity, let $R_2 = R_3$

$= 10.0\ \text{k}\Omega$, so $v_O = R_1(i_S - 4\ \text{mA})$. $i_S = 20\ \text{mA}$

$\Rightarrow v_O = 10\ \text{V} = R_1(20-4) \Rightarrow R_1 = 10/16 = 625\ \Omega$.

2.3



$v_N = \frac{r_d}{r_d+R}v_1 = \frac{2}{3}v_1$;

$v_1 = \frac{(r_d+R)\|R_1}{[(r_d+R)\|R_1]+R_2+r_o}v_T \cong \frac{1}{1+100}v_T = \frac{v_T}{101}$;

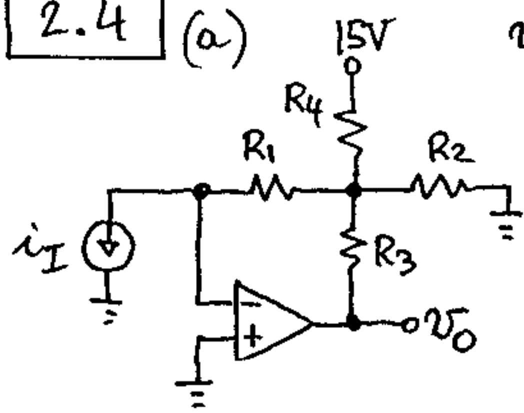
(2.2)

$\beta \cong \frac{2}{3}\times\frac{1}{101} = \frac{2}{303}$ V/V; $T = a\beta = 2\times10^5\times\frac{2}{303} = 1320.$

$A \cong A_{ideal}(1-1/T) = A_{ideal}\times(-0.9992)$;

$R_i \cong \frac{R\|r_d}{1+T} = 505\ \Omega$; $R_o \cong \frac{r_o}{1+T} = 57\ m\Omega.$

2.4 (a)



$v_O = Ri_I - 5V$; $i_I = 0 \Rightarrow v_O = -(R_3/R_4)15 = -5V \Rightarrow R_4 = 3R_3$. $R = \Delta v_O/\Delta i_I = 10/10^{-6} = 10\ M\Omega$. Let $R_1 = 1\ M\Omega$. Let X be the node where the resistors meet. Then, $i_I = 1\mu A \Rightarrow v_X = 10^6\times10^{-6} = 1V$ and $v_O = +5V$. KCL: $(5-1)/R_3 + (15-1)/R_4 = 10^{-6} + 1/R_2$. Let $R_2 = 1k\Omega$. Then, $R_3 = 8.658\ k\Omega$ (use 8.66 kΩ) and $R_4 = 25.97\ k\Omega$ (use 26.1 kΩ).

(b) $\beta \cong (R_2 /\!/ R_4)/[(R_2/\!/R_4)+R_3] = 1/11$ V/V. $100/a\beta \le 1 \Rightarrow a \ge 1{,}100$ V/V.

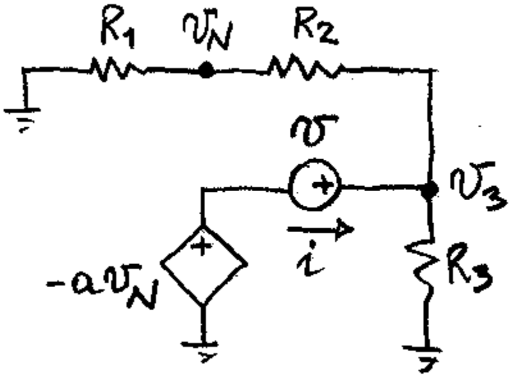
2.5 (a) $i_O = i_{R_2} + i_{R_3} = i_{R_1} + i_{R_3} = \frac{v_I}{R_1} + \left(\frac{R_2}{R_1}v_I\right)\frac{1}{R_3} = \frac{v_I}{R_1}\left(1+\frac{R_2}{R_3}\right) = \frac{v_I}{R}$, $R = \frac{R_1}{1+R_2/R_3}$.

(b) $R_i = R_1 \Rightarrow R_1 = 1\ M\Omega$. Let $R_2 = R_1 = 1\ M\Omega$. Then, for $R = (1\ V)/(1\ mA) = 10^3\ \Omega$, we need $10^3 = 10^6/(1+10^6/R_3)$, or $R_3 \cong 1\ k\Omega$.

(c) $|v_L| \le 13 - |v_I|$ V.

(2.3)

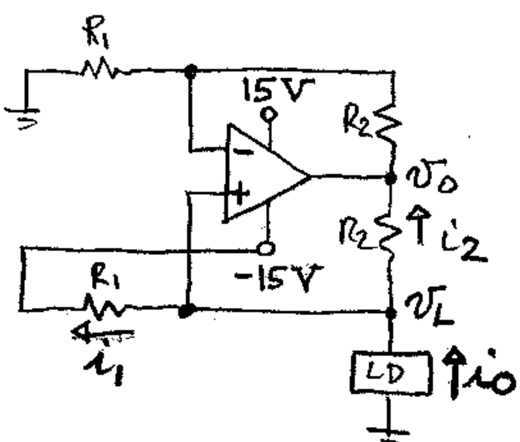
2.6



$v_3 = (1 + R_2/R_1)v_N = 1.99 v_N$; $-av_N + v = v_3$
$\Rightarrow v_N = v/(1.99 + 10^3)$; $i = v_3/(R_1 + R_2) +$
$v_3/R_3 = [1.99 v/(1.99 + 10^3)]/[(199 \| 1)10^3]$
$= v/(500{,}995)$. $R_o = v/i \cong 501\ k\Omega$.

2.7 Eq. (2.7) gives $\lim_{a \to \infty} R_o = \infty$, so (c) is correct. (a) is wrong because it ignores negative feedback. (b) is wrong because the op amp keeps a virtual short between v_N and v_P, not between v_N and v_O.

2.8



$R_1 = 15/1.5 = 10.0\ k\Omega$, 1% ; $R_2 \leq 0.3 R_1$. Use $R_2 =$ 2.00 kΩ, 1%. Then, $v_O = (1 + 2/10)v_L = 1.2\ v_L$.

(a) $v_L = -2 \times 1.5 = -3$ V ; $v_O = -3.6$ V;
$i_1 = [3 - (-15)]/10 = 1.2$ mA; $i_2 = [-3 - (-3.6)]/2 =$

(2.4)

0.3 mA; clearly, $i_0 = i_1 + i_2 = 1.2 + 0.3 = 1.5$ mA.

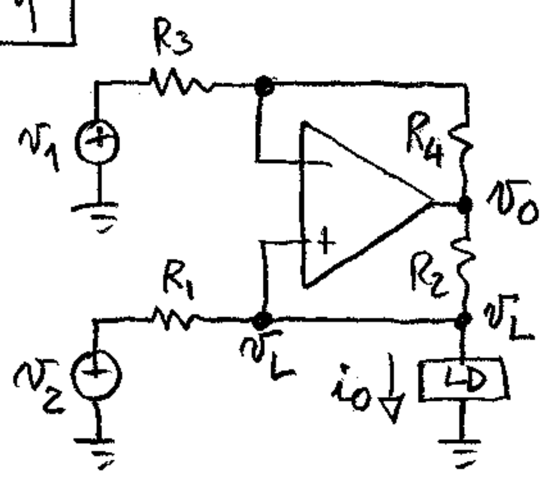
(b) $v_L = -9$ V, $v_O = -10.8$ V, $i_1 = 0.6$ mA, $i_2 = 0.9$ mA.

(c) With the cathode at ground, the zener gives $v_L = -5$ V, $v_O = -6$ V, $i_1 = 1$ mA, $i_2 = 0.5$ mA.

(d) $v_O = v_L = 0$, $i_1 = 1.5$ mA, $i_2 = 0$.

(e) With a 10-kΩ load the op amp saturates at −13 V. By KCL, $(0 - v_L)/10 = (v_L + 15)/10 + (v_L + 13)/12$, or $v_L = -80/7$ V. So, $i_0 = 1.143$ mA, $i_1 = 0.357$ mA, $i_2 = 0.786$ mA. Because of saturation, we no longer have $i_0 = 1.5$ mA.

2.9



Superposition: $v_O = -\frac{R_4}{R_3} v_1 + \left(1 + \frac{R_4}{R_3}\right) v_L$; KCL:

$$i_0 = \frac{v_2 - v_L}{R_1} + \frac{v_O - v_L}{R_2} = \frac{v_2}{R_1} + \frac{v_O}{R_2} - \left(\frac{1}{R_1} + \frac{1}{R_2}\right) v_L$$

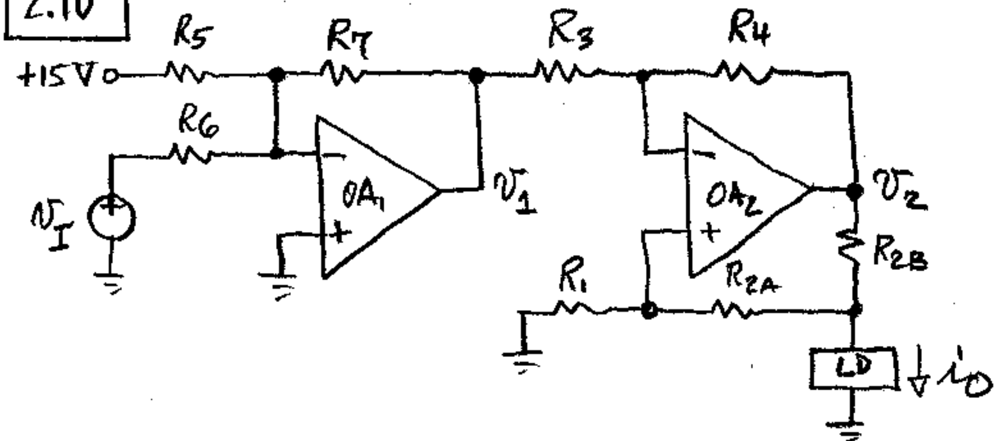
$$= \frac{v_2}{R_1} - \frac{R_4}{R_2 R_3} v_1 - v_L \left(\frac{1}{R_1} + \frac{1}{R_2} - \frac{1}{R_2} - \frac{R_4}{R_2 R_3}\right)$$

$$= \frac{1}{R_1}\left(v_2 - \frac{R_1}{R_2}\frac{R_4}{R_3} v_1\right) - \frac{v_L}{R_2}\left(\frac{R_2}{R_1} - \frac{R_4}{R_3}\right)$$

$$= \frac{1}{R_1}\left(v_2 - \frac{R_4/R_3}{R_2/R_1} v_1\right) - \frac{v_L}{R_o}, \quad R_o = \frac{R_2}{R_2/R_1 - R_4/R_3}.$$

If $R_4/R_3 = R_2/R_1$, then $i_O = \frac{1}{R_1}(v_2 - v_1)$, and $R_o = \infty$.

2.10

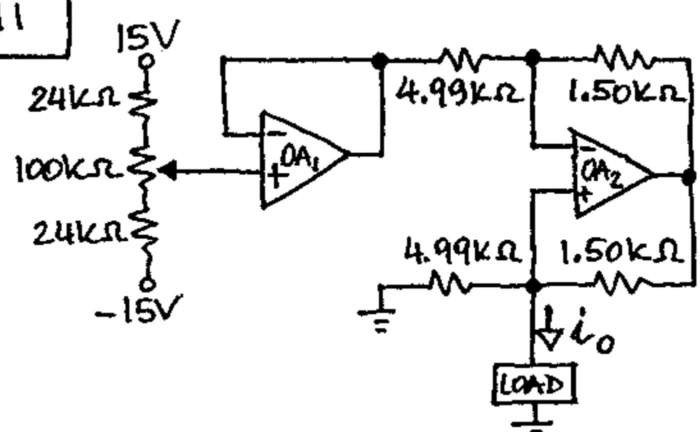


Let $R_1 = R_3 = R_4 = 10\ \text{k}\Omega$. Assume a maximum drop of 2 V across R_{2B}, so $R_{2B} = 2/20 = 100\ \Omega$. Then, $R_{2A} = 10\ \text{k}\Omega - 100\ \Omega = 9.9\ \text{k}\Omega$.

$v_I = 0 \Rightarrow v_1 = -(R_7/R_5)15 = -0.4 \Rightarrow R_5/R_7 = 37.5$

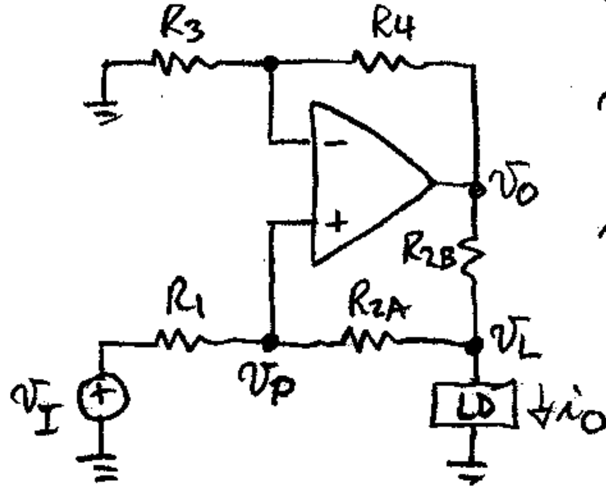
$v_I = 10\ \text{V} \Rightarrow v_1 = -0.4 - (R_7/R_6)10 = -2 \Rightarrow R_6/R_7 = 6.25$. Use $R_7 = 2\ \text{k}\Omega$, $R_6 = 12.5\ \text{k}\Omega$, $R_5 = 75\ \text{k}\Omega$.

2.11



OA_1 provides a variable voltage from −10V to +10V, which OA_2 converts to a variable current from −2mA to +2mA.

2.12 (a)



$$v_O = \left(1 + \frac{R_4}{R_3}\right) v_P$$

$$v_P = \frac{R_{2A} v_I + R_1 v_L}{R_1 + R_{2A}}$$

$$i_O = \frac{v_I - v_L}{R_1 + R_{2A}} + \frac{v_O - v_L}{R_{2B}}$$

Eliminating v_O and v_P gives

$$i_O = \frac{1}{R} v_I - \frac{1}{R_o} v_L, \text{ where}$$

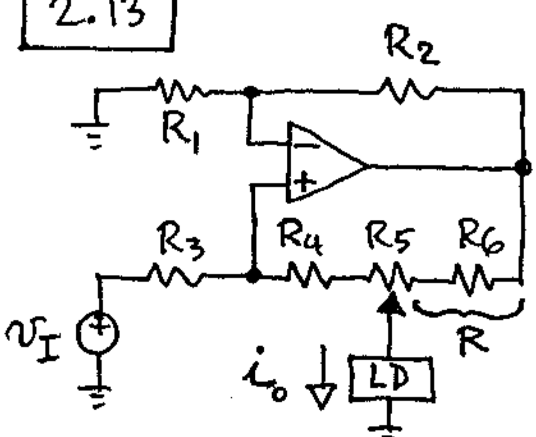
$$R = \frac{R_3 R_{2B} (R_1 + R_{2A})}{R_3 (R_{2A} + R_{2B}) + R_4 R_{2A}}, \quad R_o = \frac{R_{2B}(1 + R_{2A}/R_1)}{R_4/R_3 - (R_{2A} + R_{2B})/R_1}.$$

To make $R_o = \infty$ impose $R_4/R_3 = R_2/R_1$, where $R_2 = R_{2A} + R_{2B}$. This gives

$$R = \frac{R_3 R_{2B} (R_1 + R_{2A})}{R_4 (R_1 + R_{2A})} = \frac{R_3}{R_4} R_{2B} \Rightarrow \frac{1}{R} = \frac{R_4/R_3}{R_{2B}}.$$

(b) Imposing $10 = 13 - (R_4/R_3)10$ gives $R_4/R_3 = 0.3$. Let $R_1 = R_3 = 100\ \text{k}\Omega$, $R_4 = R_{2A} + R_{2B} = 30.1\ \text{k}\Omega$. Then, imposing $(R_4/R_3)/R_{2B} = 0.301/R_{2B} = 1\ \text{mA/V}$ gives $R_{2B} = 301\ \Omega$. Finally, $R_{2A} = 30.1 - 0.301 = 29.8\ \text{k}\Omega$ (use $30.1\ \text{k}\Omega$, 1%).

2.13



$$i_o = \frac{R_2/R_1}{R} v_I$$

Wiper to the right:

$$\frac{R_2/R_1}{R_6} = \frac{1}{10^3}.$$

Wiper to the left:

(2.7)

$\frac{R_2/R_1}{R_5+R_6} = \frac{1}{10^4}$. Let $R_5 = 10$-kΩ pot. Substituting and solving yields $R_2/R_1 = 10/9$ and $R_6 = 10/9$ kΩ. Use $R_1 = 90.9$ kΩ, $R_2 = 100$ kΩ, $R_3 = 90.9$ kΩ, $R_5 = 10.0$ kΩ, $R_6 = 1.10$ kΩ, $R_4 = 100 - 10 - 1.1 = 88.7$ kΩ, all 1%.

2.14 (a) Denote the output of OA_1 as v_{O1}, and that of OA_2 as v_{O2}. By inspection, we have $v_{O2} = v_L$. By the superposition principle,

$$v_{O1} = -\frac{R_4}{R_3}v_1 + \left(1+\frac{R_4}{R_3}\right)\frac{R_2 v_2 + R_1 v_L}{R_1+R_2}$$

$$= \frac{1+R_4/R_3}{1+R_1/R_2}v_2 - \frac{R_4}{R_3}v_1 + \frac{1+R_4/R_3}{1+R_2/R_1}v_L .$$

$$i_O = \frac{v_{O1}-v_L}{R_5} = A_2 v_2 - A_1 v_1 - \frac{1}{R_o}v_L, \text{ where}$$

$$A_2 = \frac{1+R_4/R_3}{1+R_1/R_2}\frac{1}{R_5},\ A_1 = \frac{R_4}{R_3}\frac{1}{R_5}, \text{ and}$$

$$\frac{1}{R_o} = \frac{1}{R_5}\left(1 - \frac{1+R_4/R_3}{1+R_2/R_1}\right) = \frac{1}{(1+R_2/R_1)R_5}\left(\frac{R_2}{R_1} - \frac{R_4}{R_3}\right)$$

To make $R_o \to \infty$ impose $R_4/R_3 = R_2/R_1$, after which it is readily seen that $A_1 = A_2 = \frac{R_2/R_1}{R_5}$.
In summary, imposing $R_4/R_3 = R_2/R_1$ gives $i_O = Av_I - \frac{1}{R_o}v_L$, $A = \frac{R_2/R_1}{R_5}$, $v_I = v_2 - v_1$, $R_o = \infty$.

(b) If the resistances are mismatched, A_1 and A_2 will also be mismatched, so we no longer have true difference operation.

(2.8)

Writing $R_o = \frac{(1+R_2/R_1)R_5}{R_2/R_1 - (R_2/R_1)(1-\varepsilon)} = \left(1+\frac{R_2}{R_1}\right)\frac{R_5}{\varepsilon}$

gives, for 1% resistors, $|R_o| \geq 25(1+R_2/R_1)R_5$.

2.15 (a) Denote the output of OA_1 as v_{O1}, that of OA_2 as v_{O2}. By OA_2's action, $v_{O1} = v_L$ and $v_{O2} = v_L + R_5 i_O$. By the superposition principle,

$$v_{O1} = -\frac{R_4}{R_3}v_I + \left(1+\frac{R_4}{R_3}\right)\frac{R_1}{R_1+R_2}(v_L + R_5 i_O) = v_L.$$

Solving for i_O gives $i_O = Av_I - (1/R_o)v_L$,

$$A = \frac{1+R_2/R_1}{1+R_4/R_3}\,\frac{R_4/R_3}{R_5},\quad R_o = \frac{(1+R_4/R_3)R_5}{R_2/R_1 - R_4/R_3}.$$

Imposing $R_4/R_3 = R_2/R_1$ gives $R_o = \infty$ and $A = (R_2/R_1)/R_5$.

(b) Writing $R_o = \frac{(1+R_2/R_1)R_5}{R_2/R_1 - (R_2/R_1)(1-\varepsilon)}$

$= (1+R_1/R_2)R_5/\varepsilon$. With 1% resistors we can expect $|R_o| \geq 25(1+R_1/R_2)R_5$.

2.16 (a) Denote the outputs of OA_1 and OA_2 as v_{O1} and v_{O2}. We have $v_{O2} = -v_{O1} = -[-v_I - (R_1/R_2)v_L] = v_I + (R_1/R_2)v_L$; $i_O =$

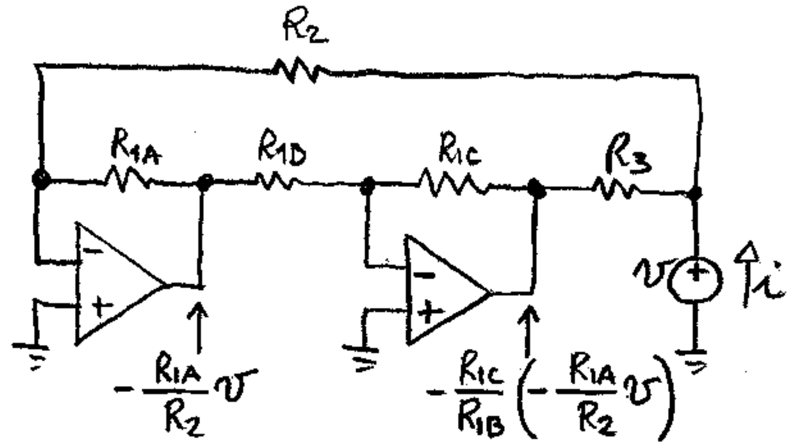
$$\frac{v_{O2}-v_L}{R_3} - \frac{v_L}{R_2} = \frac{v_I}{R_3} - v_L\left[\frac{1}{R_3} + \frac{1}{R_2} - \frac{R_1/R_2}{R_3}\right],$$

or $i_O = Av_I - \frac{1}{R_o}v_L$, $A = \frac{1}{R_3}$, $R_o = \frac{R_2R_3}{R_2+R_3-R_1}$.

To achieve $R_o = \infty$, impose $R_2 + R_3 = R_1$.

(2.9)

(b) To find the effect of mismatches upon R_o, apply a test voltage at the output, as shown:



$$i = \frac{v}{R_2} + \frac{v - (R_{1A}R_{1C}/R_2R_{1B})v}{R_3}$$

$$= v\left[\frac{1}{R_2} + \frac{1}{R_3} - \frac{R_{1A}(R_{1C}/R_{1B})}{R_2R_3}\right]$$

$$R_o = \frac{v}{i} = \frac{R_2R_3}{R_2 + R_3 - R_{1A}(R_{1C}/R_{1B})}.$$

R_o is maximized when R_2, R_3, and R_{1B} are maximized, and R_{1A} and R_{1C} are minimized. For 1% resistors, rewrite as

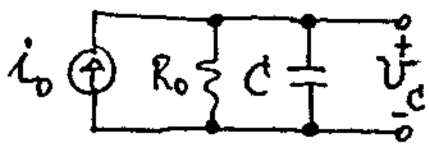
$$R_{o(max)} = \frac{(R_2 \times R_3)1.01^2}{(R_2+R_3)1.01 - (R_2+R_3)0.99\,(0.99/1.01)}$$

$$\cong 25\frac{R_3}{1+R_3/R_2}.$$

2.17 (a) $i_O = 1\,mA$, $R_o = \infty$. $v_C(t) = (i_O/C)t = (10^{-3}/10^{-7})t = 10^4 t$. $v_O(t) = (1 + R_2/R_1)v_C(t) = 1.295 \times 10^4 t$. A linear ramp.

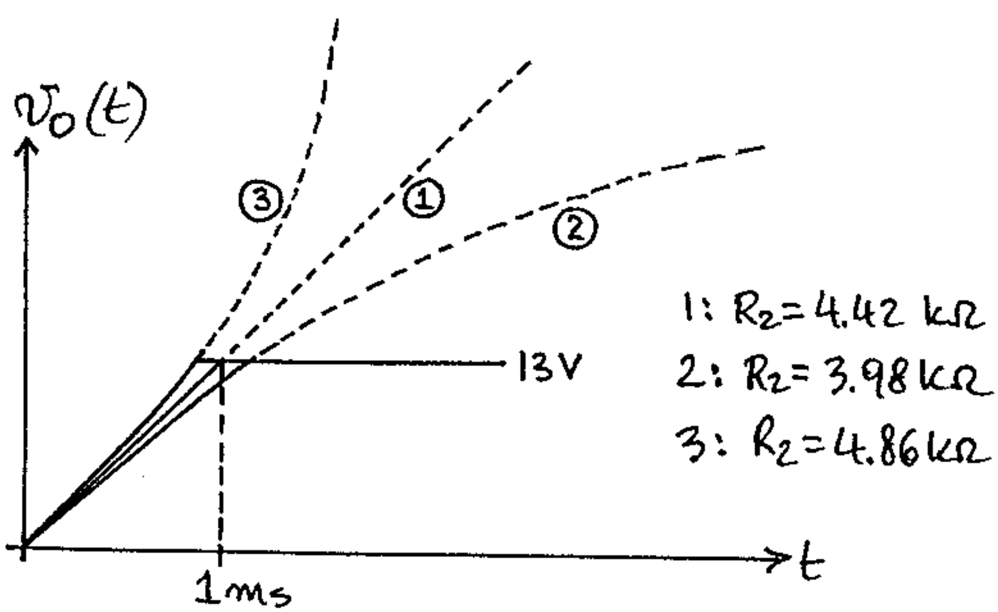
(b) $13 = 1.295 \times 10^4 t \Rightarrow t \cong 1\,ms$.

2.18 The capacitor sees the equivalent circuit on the left.



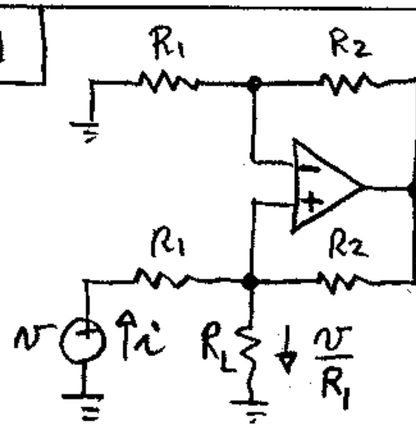
(a) $i_o = 1\,mA$; $R_o = R_2/(R_2/R_1 - R_4/R_3) = 4.42/(4.42/15 - 3.978/15) = 150\,k\Omega$; $R_o i_o = 150 \times 1 = 150\,V$; $\tau = R_o C = 150 \times 10^3 \times 10^{-7} = 15\,ms$; $(1 + R_2/R_1)R_o i_o = (1 + 3.980/15)150 \cong 190\,V$. $v_O(t) = 190\,V\,[1 - \exp(-t/15\,ms)]$. $13 = 190 \times [1 - \exp(-t/15\,ms)] \Rightarrow t = 1.06\,ms$.

(b) Now $R_o = -150\,k\Omega$, $(1 + R_2/R_1) \times |R_o| i_o \cong 200\,V$, $v_O(t) = 200\,V\,[\exp(t/15\,ms) - 1]$. $13 = 200\,[\exp(t/15\,ms) - 1] \Rightarrow t = 0.95\,ms$.



(2.11)

2.19

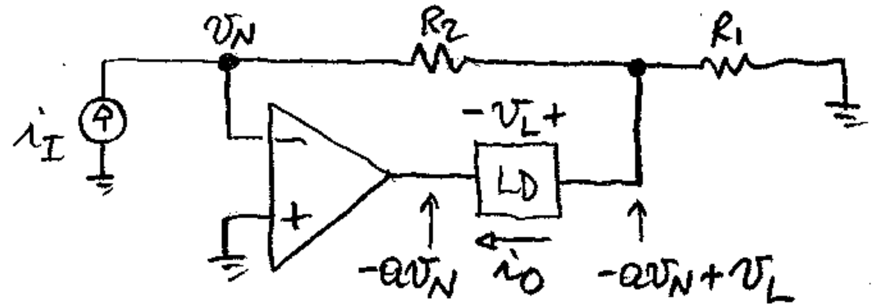


$$i = \frac{v - (v/R_1)R_L}{R_1} = \frac{v}{R_1}\left(1 - \frac{R_L}{R_1}\right);$$

$$R_i = \frac{v}{i} = \frac{R_1}{1 - R_L/R_1}.$$

$R_L < R_1 \Rightarrow R_o > 0$; $R_L < R_1 \Rightarrow R_i < 0$; $R_L = R_1 \Rightarrow R_i = \infty$.

2.20 (a)



Ω: $v_N - (-av_N + v_L) = R_2 i_I$

$\Rightarrow v_N = (R_2 i_I + v_L)/(1+a)$. KCL:

$$i_O = i_I + \frac{av_N - v_L}{R_1} = i_I + \frac{a(R_2 i_I + v_L)}{(1+a)R_1} - \frac{v_L}{R_1}$$

$$= i_I\left(1 + \frac{R_2/R_1}{1+1/a}\right) - \frac{1}{R_1}v_L\left(1 - \frac{1}{1+a}\right) = Ai_I - \frac{v_L}{R_o},$$

$A = 1 + \dfrac{R_2/R_1}{1+1/a}$, $R_o = R_1(1+a)$

(b) Use $R_1 = 2\ \text{k}\Omega$, $R_2 = 18\ \text{k}\Omega$.

$A_{ideal} = 10$ A/A; $A_{actual} = 1 + 9/(1 + 1/200{,}000) = 9.999955$; gain error $= -0.00045\%$.

$R_o \cong 2 \times 10^3 \times (1 + 200{,}000) = 400\ \text{M}\Omega$.

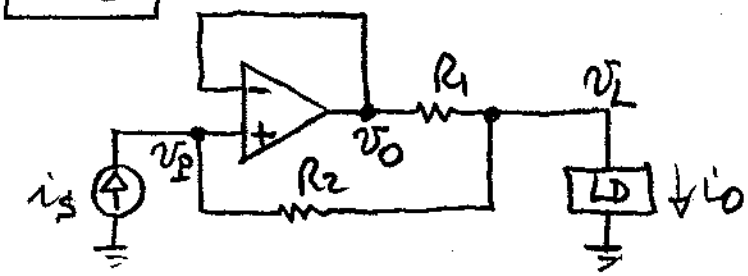
2.21 The op amp keeps $v_O = v_N = v_P$. By the superposition principle, $v_P = (R_s // R_2) i_S + \frac{R_s}{R_s + R_2} v_L$. By KCL, $i_O = (v_P - v_L)/(R_1 // R_2)$. Substituting,

$$i_O = \frac{R_s // R_2}{R_1 // R_2} i_S - \frac{v_L}{R_1 // R_2}\left[1 - \frac{R_s}{R_s + R_2}\right] = A i_S - \frac{v_L}{R_o},$$

$$A = \frac{1 + R_2/R_1}{1 + R_2/R_s}, \quad R_o = \frac{R_s + R_2}{1 + R_2/R_1}.$$

For $R_s \to \infty$ we get $A = 1 + R_2/R_1$ and $R_o = \infty$.

2.22



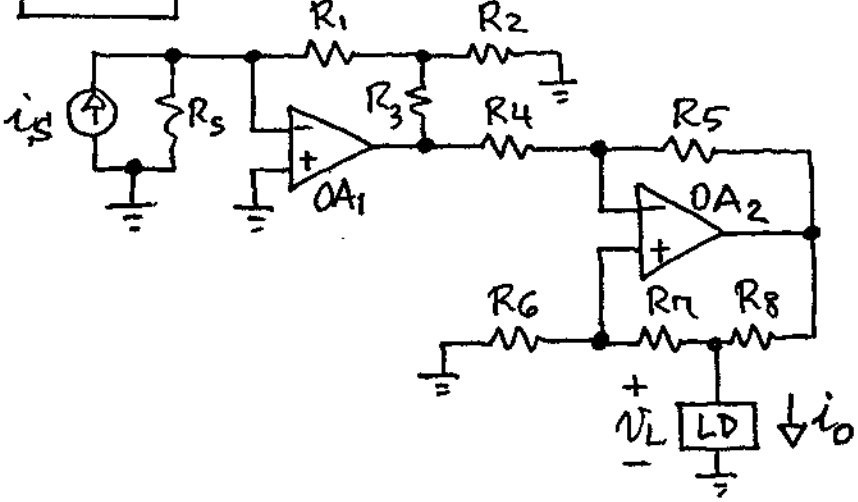
$v_P = v_L + R_2 i_S$; $v_O = a(v_P - v_O) \Rightarrow v_O = \frac{a}{1+a} v_P$

$v_O = \frac{a}{1+a}(v_L + R_2 i_S)$. $i_O = i_S + \frac{v_O - v_L}{R_1} \Rightarrow$

$$i_O = i_S + \frac{1}{R_1}\left[\frac{a}{1+a} v_L - v_L + \frac{a}{1+a} R_2 i_S\right] = A i_S - \frac{1}{R_o} v_L,$$

$A = 1 + (R_2/R_1)/(1 + 1/a)$, $R_o = R_1(1+a)$.

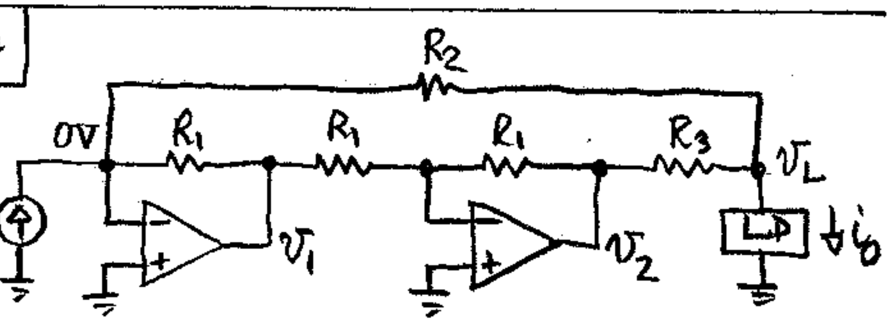
2.23



2.13

Choose the I-V converter components for a 10-V full scale at OA_1's output. Thus, let $R_1 = 1\,M\Omega$, $R_2 = 1\,k\Omega$, $R_3 = 100\,k\Omega$. $i_{O(max)} = 10^5 \times 100 \times 10^{-9} = 10\,mA$. Imposing $R_E = 100\,\Omega$ yields a voltage compliance of $13 - 0.1\times10 = 12\,V > 5\,V$. Finally, let $R_4 = R_5 = R_6 = 100\,k\Omega$, $R_7 = 99.9\,k\Omega$.

2.24



$v_1 = -R_1 i_I - (R_1/R_2)v_L$; $v_2 = -v_1 = R_1 i_I + \frac{R_1}{R_2}v_L$.

$$i_O = \frac{v_2 - v_L}{R_3} - \frac{v_L}{R_2} = \frac{R_1}{R_3} i_I - v_L\left[\frac{1}{R_2} + \frac{1}{R_3} - \frac{R_1}{R_2 R_3}\right].$$

Impose $R_2 + R_3 = R_1$ to achieve $R_o = \infty$, and $R_1/R_3 = 10$ to achieve the desired gain. Moreover, $R_i = 0$ because of the input virtual ground. Use $R_1 = 10.0\,k\Omega$, $R_3 = 1.00\,k\Omega$, and $R_2 = 9.09\,k\Omega$.

2.25 The output of OA_1 is $v_1 = -\frac{R}{R_2}v_2 - \frac{R}{R_4}v_4$.

By the superposition principle,

$$v_O = -\frac{R_F}{R_1}v_1 - \frac{R_F}{R_3}v_3 - \frac{R_F}{R}v_1 =$$

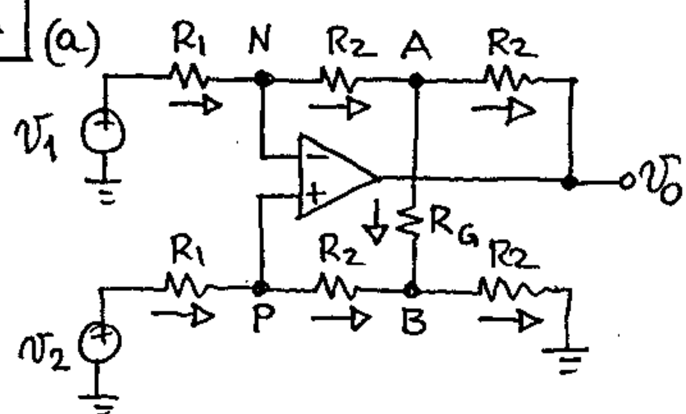
$$= \frac{R_F}{R_2}v_2 + \frac{R_F}{R_4}v_4 - \frac{R_F}{R_1}v_1 - \frac{R_F}{R_3}v_3.$$

(2.14)

The circuit sums the even-numbered inputs with positive gains, and the odd-numbered inputs with negative gains. Since the summing junctions of both op amps are at virtual ground, leaving an input floating has no effect. By contrast, leaving any input floating in Fig. P1.31 affects the output because in general $v_N = v_P \neq 0$.

2.26 Applying a test voltage v in Fig. 2.14(a) yields, by the virtual short concept, $i = v/(R_1 + 0 + R_1) = v/2R_1$. Hence, $R_{id} = 2R_1$. In Fig. 2.14(b) both resistances R_1 carry the same current. Hence, applying a test voltage v yields $i = 2i_{R_1} = 2v/(R_1+R_2)$. Consequently, $R_{ic} = (R_1+R_2)/2$.

2.27 (a)



KCL at N: $\dfrac{v_1 - v_N}{R_1} = \dfrac{v_N - v_A}{R_2}$

KCL at P: $\dfrac{v_2 - v_P}{R_1} = \dfrac{v_P - v_B}{R_2}$.

(2.15)

Letting $v_N = v_P$ and subtracting,

$v_A - v_B = (R_2/R_1)(v_2 - v_1)$ .... (1)

KCL at A: $\frac{v_1 - v_A}{R_1+R_2} = \frac{v_A - v_B}{R_G} + \frac{v_A - v_O}{R_2}$.

KCL at B: $\frac{v_2 - v_B}{R_1+R_2} + \frac{v_A - v_B}{R_G} = \frac{v_B}{R_2}$. Subtracting,

$$\frac{(v_2 - v_1) + (v_A - v_B)}{R_1+R_2} + 2\frac{v_A - v_B}{R_G} = \frac{(v_B - v_A) + v_O}{R_2}.$$

Combining with Eq. (1),

$$(v_2 - v_1)\left(\frac{1 + R_2/R_1}{R_1+R_2} + 2\frac{R_2/R_1}{R_G} + \frac{1}{R_1}\right) = \frac{v_O}{R_2}.$$

Solving for v_O and simplifying,

$$v_O = 2\frac{R_2}{R_1}\left(1 + \frac{R_2}{R_G}\right)(v_2 - v_1).$$

(b) Let $R_G = 100\text{-k}\Omega$ pot in series with a $5\text{k}\Omega$ resistor. Then, $100 = 2(R_2/R_1)(1+R_2/5)$ and $10 = 2(R_2/R_1)[1 + R_2/(100+5)]$. Dividing, $100/10 = (1+R_2/5)/(1+R_2/105)$. Solving, $R_2 = 85.9\text{ k}\Omega$. Back substituting yields $R_1 = 31.24\text{ k}\Omega$. Use $R_1 = 31.6\text{ k}\Omega$, $R_2 = 86.6\text{ k}\Omega$, $R_G = 100\text{-k}\Omega$ pot + $4.99\text{ k}\Omega$, all 1%.

2.28 (a) $v_{O2} = -(R_3/R_G)v_O$. Superposition: $v_{P1} = \frac{R_2 v_2 + R_1 v_{O2}}{R_1+R_2}$. Voltage divider: $v_{N1} = [R_2/(R_1+R_2)]v_1$. Eliminating v_{O2} and letting $v_{N1} = v_{P1}$ gives $v_O = \frac{R_2}{R_1}\frac{R_G}{R_3}(v_2 - v_1)$.

(2.16)

(b) Let $R_1 = R_2 = 10\,k\Omega$. Then, $A = R_G/R_3$. Let $R_3 = 1\,k\Omega$ and let R_G be a 100-kΩ pot in series with a 1-kΩ resistor. Then,
$A_{(min)} = 1$ V/V, $A_{(max)} = (1+100)/1 \cong 100$ V/V.

2.29 (a) $(v_1+v_2)/2 = 10 \cos 2\pi 60t$ V; $v_2 - v_1 = 0.01 \cos 2\pi 10^3 t$ V; $A_{dm} = 2/0.01 = 200$ V/V; $A_{cm} = 0.1/10 = 0.01$ V/V; CMRR = $20 \log_{10}(200/0.01) = 86$ dB.

(b) $(v_1+v_2)/2 = 10.005 \cos 2\pi 60t$ V; $v_2 - v_1 = -0.01 \sin 2\pi 60t + 0.01 \sin 2\pi 10^3 t$ V; $A_{dm} = 2.5/0.01 = 250$ V/V. At 60 Hz, we have $0.5 = 250 \times (-0.01) + A_{cm} \times 10.005$, or $A_{cm} \cong 0.3$ V/V; CMRR = $20 \log_{10}(250/0.3) = 58.4$ dB.

2.30 $A_{dm} \cong (100\,k\Omega)/(1\,k\Omega) = 100$ V/V = 40 dB. To find A_{dm}, tie the inputs together and apply a common signal. Then,

$$A_{cm} = -\frac{99.7}{1.01} + \left(1 + \frac{99.7}{1.01}\right)\frac{102}{102 + 0.995} = 0.0367 \text{ V/V}$$

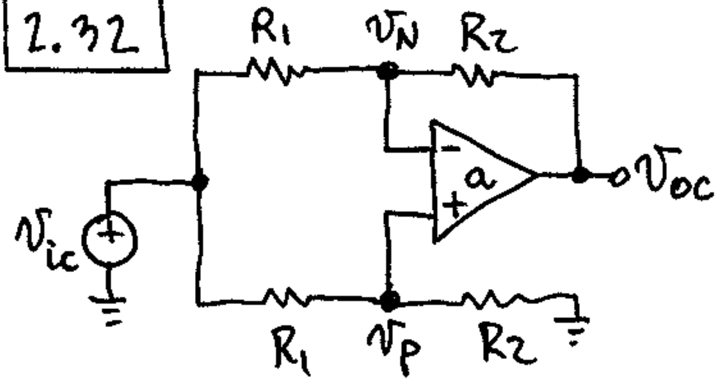
= −28.7 dB. CMRR ≅ 40 − (−28.7) = 68.7 dB.

2.31 $|A_{dm}| = 10^3$ V/V; CMRR = 10^5; $|A_{cm}| = 10^{-2}$ V/V.

$v_{id} = v_2 - v_1 = 2$ mV; $v_{ic} = (v_1+v_2)/2 = 4$ V;
$|v_{od}| = 10^3 \times 2 \times 10^{-3} = 2$ V; $|v_{oc}| = 10^{-2} \times 4 = 0.04$ V.
Error = $100|v_{oc}|/|v_{od}| = 2\%$.

(2.17)

2.32



$$v_{OC} = a(v_P - v_N) = a\left[\frac{R_2}{R_1+R_2}v_{IC} - \frac{R_2 v_{IC} + R_1 v_{OC}}{R_1+R_2}\right]$$

$$= a\left[\frac{R_2}{R_1+R_2}v_{IC} - \frac{R_2}{R_1+R_2}v_{IC} - \frac{R_1}{R_1+R_2}v_{OC}\right]$$

$\Rightarrow v_{OC}(1+a\beta) = 0 \Rightarrow v_{OC} = 0$ regardless of v_{IC}.

$\Rightarrow$ CMRR $= \infty$. Intuitively: v_{OC} can only be zero. Suppose v_{OC} was positive. Then, v_N would be $> v_N$, implying $v_O = a(v_P - v_N) < 0$, a contradiction.

2.33 $v_{N1} = v_{P1} = v_1 = 5\text{ V} - 5 \sin\omega t \text{ mV}$;

$v_{N2} = v_{P2} = v_2 = 5\text{V} + 5 \sin\omega t \text{ mV}$;

$$v_{O1} = v_{N1} + R_3\frac{v_{N1} - v_{N2}}{R_G} = 5\text{V} - 5 \sin\omega t \text{ mV} +$$

$$\frac{10^6}{2\times10^3}(-10 \sin\omega t \text{ mV}) = 5\text{V} - 5.005 \sin\omega t \text{ V};$$

$v_{O2} = 5\text{ V} + 5.005 \sin\omega t \text{ V}$;

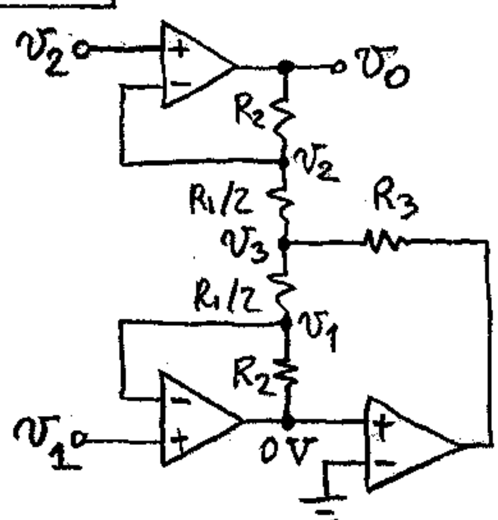
$$v_{N3} = v_{P3} = \frac{R_2}{R_1+R_2}v_{O2} = 2.5\text{ V} + 2.5025 \sin\omega t \text{ V};$$

$$v_O = \frac{R_2}{R_1}(v_{O2} - v_{O1}) = 10.01 \sin\omega t \text{ V}.$$

2.34 $v_O = v_{O1} - v_{O2} = a_1(v_{P1} - v_{N1}) - a_2(v_{P2} - v_{N2}) = a[(v_{P1} - v_{P2}) - (v_{N1} - v_{N2})] = a[v_I - R_G v_O/(R_G + 2R_3)]$. This is of the type $v_O = a(v_I - \beta v_O)$, $\beta = R_G/(R_G + 2R_3)$.

2.35 From Problem 2.34, $\beta_I = 1/A_I = 1/50$ V/V; moreover, $\beta_{II} = 1/A_{II} = 1/20$ V/V. We can guarantee a 0.1% maximum deviation of $A = A_I \times A_{II}$ from ideality by imposing a 0.05% maximum deviation of A_I and A_{II}. Thus, $100/a_I\beta_I \le 0.05 \Rightarrow a_I \geq 100 \times 50/0.05 = 10^5$ V/V; likewise, $a_{II} \geq 4 \times 10^4$ V/V.

2.36



$v_1 = \dfrac{R_2}{R_2 + R_1/2} v_3 \Rightarrow$

$v_3 = \left(1 + \dfrac{R_1}{2R_2}\right) v_1$;

$\dfrac{v_O - v_2}{R_2} = \dfrac{v_2 - v_3}{R_1/2}$.

Eliminating v_3,

$v_O = \left(1 + \dfrac{2R_2}{R_1}\right)(v_2 - v_1)$.

2.37 (a) Superposition:

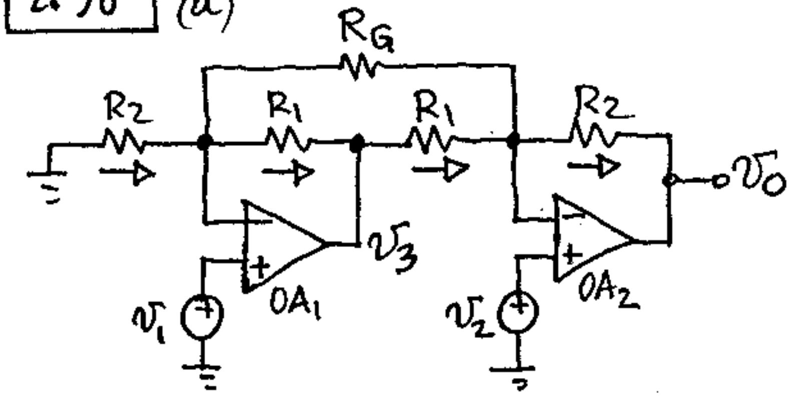
$$v_O = \left[1 + \frac{R_2}{R_1}\right]\left[v_{CM} + \frac{v_{DM}}{2}\right] - \frac{R_2}{R_1}\left[1 + \frac{R_1}{R_2}(1-\varepsilon)\right]\left[v_{CM} - \frac{v_{DM}}{2}\right]$$

$$= \left(1 + \frac{R_2}{R_1} - \frac{\varepsilon}{2}\right) v_{DM} + \varepsilon v_{CM}$$

(2.19)

(b) With 1% resistors, ε can be as large as 0.04. Since this is much less than 100, we can write $CMRR \geq 20\log_{10}(100/0.04) = 68$ dB.

2.38 (a)



$v_{N1} = v_{P1} = v_1$, $v_{N2} = v_{P2} = v_2$. Applying KCL:

$$\frac{0-v_1}{R_2} = \frac{v_1-v_2}{R_G} + \frac{v_1-v_3}{R_1}; \quad \frac{v_2-v_O}{R_2} = \frac{v_1-v_2}{R_G} + \frac{v_3-v_2}{R_1}.$$

Adding the two equations pairwise gives

$$\frac{v_2-v_1}{R_2} - \frac{v_O}{R_2} = 2\frac{v_1-v_2}{R_G} + \frac{v_1-v_2}{R_1}.$$

Solving for v_O yields $v_O = \left(1+\frac{R_2}{R_1}+2\frac{R_2}{R_G}\right)(v_2-v_1)$.

(b) Let $R_G = R_{GA}+R_{GB}$, where $R_{GA} =$ 10-kΩ pot. Arbitrarily impose $R_2/R_1 = 1$, so that $A = 2(1+R_2/R_G)$. $10 \leq A \leq 100 \Rightarrow$ $5 \leq (1+R_2/R_G) \leq 50 \Rightarrow 4 \leq R_2/R_G \leq 49$. $R_G = 0 + R_{GB} \Rightarrow R_2/R_{GB} = 49$; $R_G = 10 + R_{GB}$ $\Rightarrow R_2/(10+R_{GB}) = 4$. Solving, $R_{GB} = 889\ \Omega$ (use 887 Ω, 1%); $R_2 = 49 R_{GB} = 43.5$ kΩ $=$ R_1 (use $R_1 = R_2 = 43.2$ kΩ, 1%).

(2.20)

2.39 (a) The op amps keep $v_{P1} = v_{N1} = v_1$, $v_{N2} = v_{P2} = v_2$. Let v_3 be the output of OA_2. Summing currents at v_{P1} and v_{N2} gives

$$\frac{v_O - v_2}{R} + \frac{v_1 - v_2}{R_G} + \frac{v_3 - v_2}{R} = 0$$

$$\frac{v_3 - v_1}{R} + \frac{v_2 - v_1}{R_G} + \frac{0 - v_1}{R} = 0$$

Eliminating v_3 and collecting gives

$$v_O = 2\left(1 + \frac{R}{R_G}\right)(v_2 - v_1).$$

(b) Let R_G be a 10-kΩ pot in series with a resistance R_s. Then,

$$2\left(1 + \frac{R}{R_s}\right) = 100 \Rightarrow R = 49 R_s$$

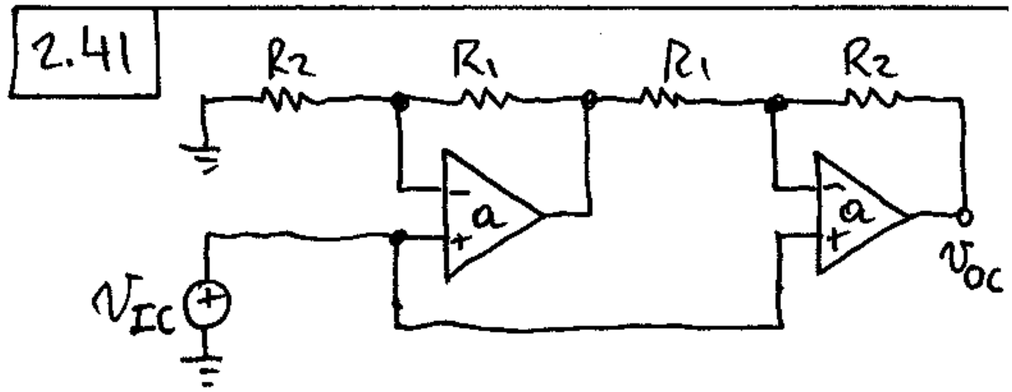
$$2\left(1 + \frac{R}{10 + R_s}\right) = 10 \Rightarrow R = 4(10 + R_s).$$ Solving,

$R_s = 888\ \Omega$ (use 887 Ω, 1%), $R = 43.5$ kΩ (use 43.2 kΩ, 1%).

2.40 Regard the capacitor as an open circuit in dc analysis. By op amp action, $v_{P1} = v_{N1} = v_1$, $v_{N2} = v_{P2} = v_2$. Moreover, the output of OA_2 is $v_3 = (1 + R_1/R_2)v_1 = -\frac{R_1}{R_2} v_O + \left(1 + \frac{R_1}{R_2}\right) v_2$. Thus, $v_O = \left(1 + \frac{R_2}{R_1}\right)(v_2 - v_1)$.

(2.21)

2.41



$$A = 1 + R_2/R_1 \Rightarrow R_2/R_1 = A-1, \quad R_1/R_2 = 1/(A-1)$$

$$v_1 = \frac{1 + R_1/R_2}{1 + \frac{1+R_1/R_2}{a}} v_{IC} = \frac{A}{A-1+A/a} v_{IC}$$

$$v_{OC} = \frac{1}{1 + \frac{1+R_2/R_1}{a}} \left[\left(1 + \frac{R_2}{R_1}\right) v_{IC} - \frac{R_2}{R_1} v_1 \right]$$

$$= \frac{A}{1+A/a} \left[1 - \frac{A-1}{A-1+A/a} \right] v_{IC}$$

$$= \frac{A}{A(1+1/a) + a(1-1/A)} v_{IC} = A_{cm} v_{IC}$$

$$CMRR = \frac{A}{A_{cm}} = a\left(1 - \frac{1}{A}\right) + A\left(1 + \frac{1}{a}\right) \cong \frac{A-1}{A} a$$

Since in general $A \ll a$. We readily see that for sufficiently large gains, or $A \gg 1$, we have $CMRR \cong a$, regardless of A.

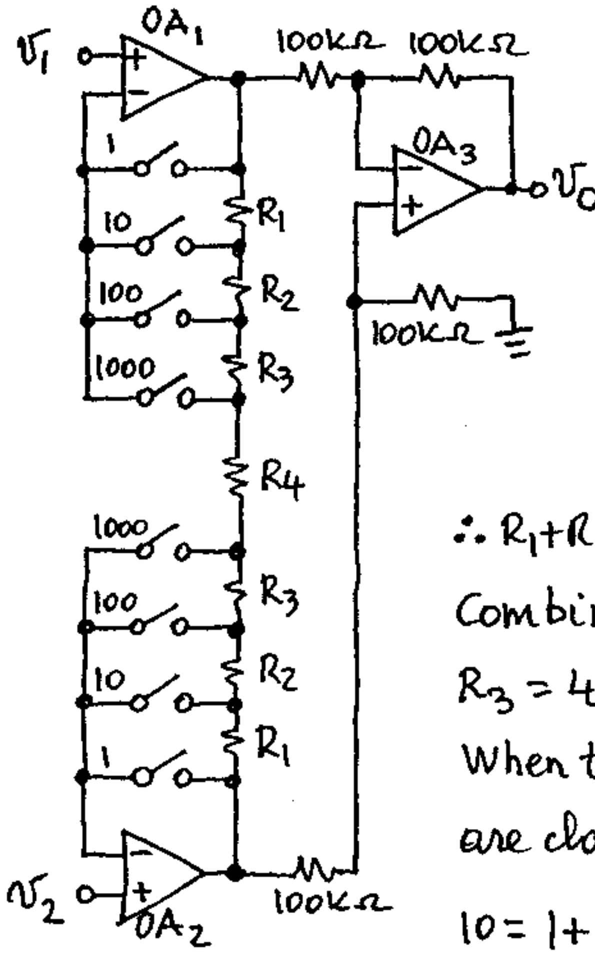
$$A = 10^3 \text{ V/V} \Rightarrow CMRR_{dB} = \left| \frac{999}{1000} 10^5 \right|_{dB} = 99.99 \text{ dB}$$

$$A = 10 \text{ V/V} \Rightarrow CMRR_{dB} = \left| \frac{9}{10} 10^5 \right|_{dB} = 99.08 \text{ dB},$$

indicating an insignificant change.

(2,22)

2.42 When the "1000" switches are closed,



$$1000 = 1 + 2\frac{R_1+R_2+R_3}{1}.$$

$\therefore R_1+R_2+R_3 = 499.5\,k\Omega.$

When the "100" switches are closed,

$$100 = 1 + 2\frac{R_1+R_2}{2R_3+1}.$$

$\therefore R_1+R_2 = 99R_3 + 49.5\,k\Omega.$

Combining yields $R_3 = 4.5\,k\Omega.$

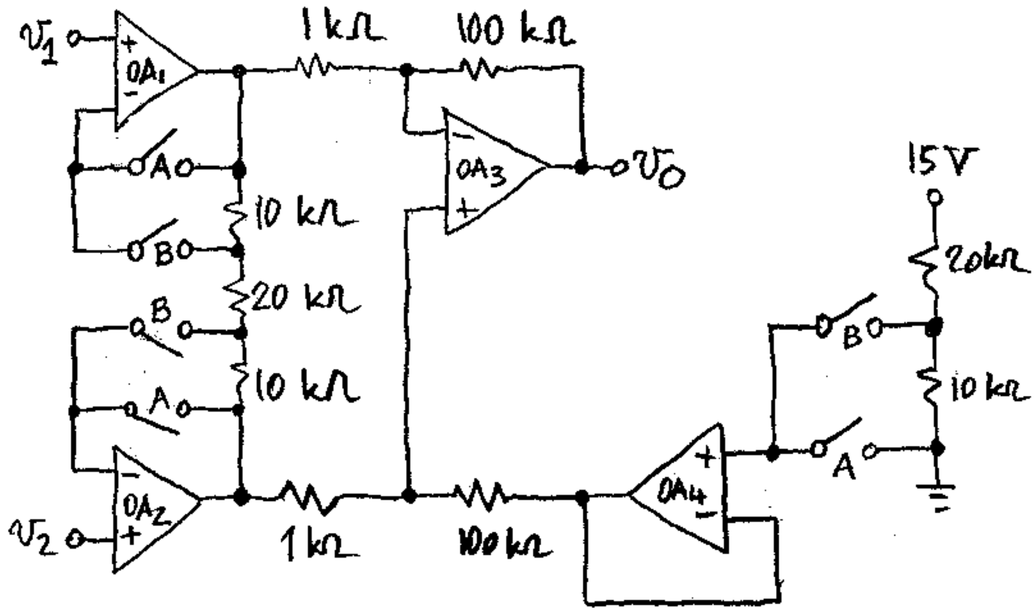
When the "10" switches are closed,

$$10 = 1 + 2\frac{R_1}{2R_2+2R_3+1}.$$

$\therefore R_1 = 9R_2 + 45\,k\Omega$. Combining yields $R_2 = 45\,k\Omega$ and $R_1 = 450\,k\Omega$. Summarizing, $R_1 = 450\,k\Omega, R_2 = 45\,k\Omega, R_3 = 4.5\,k\Omega, R_4 = 1\,k\Omega.$ All other resistors $= 100\,k\Omega.$

(2.13)

2.43



"A" switches closed ⇒ $v_O = 1 \times 100\,(v_2 - v_1) + 0$ V.

"B" switches closed ⇒ $v_O = \left(1 + 2\frac{10}{20}\right) \times 100\,(v_2 - v_1) + 5$ V.

2.44 (a) Let the outputs of OA_1 and OA_2 be v_{01} and v_{02}. Superposition:

$$v_{01} = \left(1 + \frac{R_1}{R_3}\right)v_1 - \frac{R_1}{R_3}v_L$$

$$v_{02} = \left(1 + \frac{R_5}{R_4}\right)v_2 - \frac{R_5}{R_4}\left[\left(1 + \frac{R_1}{R_3}\right)v_1 - \frac{R_1}{R_3}v_L\right]$$

KCL: $i_O = \frac{v_1 - v_L}{R_3} + \frac{v_{02} - v_L}{R_2}$. Eliminating v_{02},

$$i_O = \frac{v_2}{R_2}\left[1 + \frac{R_5}{R_4}\right] - \frac{v_1}{R_2}\left[\frac{R_5}{R_4}\left(1 + \frac{R_1}{R_3}\right) - \frac{R_2}{R_3}\right] - v_L \times \frac{R_2 + R_3 - R_1R_5/R_4}{R_2R_3}.$$

It is readily seen that imposing $R_2 + R_3 = R_1R_5/R_4$ gives

$$i_O = \frac{1}{R}(v_2 - v_1), \quad \frac{1}{R} = \frac{1 + R_5/R_4}{R_2}.$$

(b) Use $R_1 = R_4 = R_5 = 100$ kΩ, $R_2 = 2.00$

(2.24)

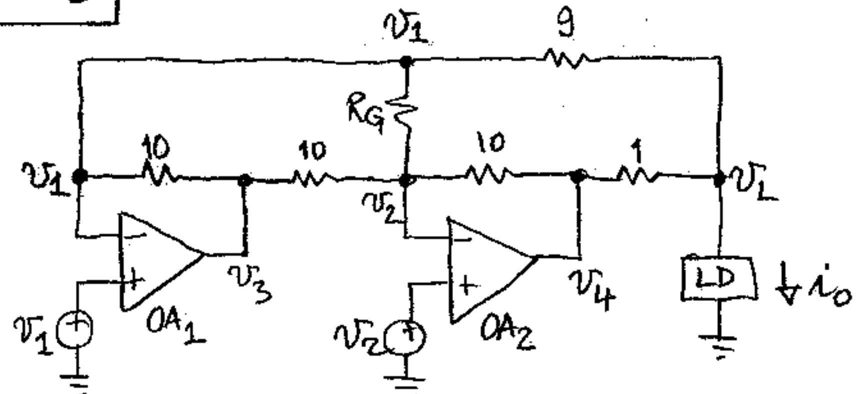
kΩ, and $R_3 = 100 - 2 = 98.0$ kΩ.

(c) If the resistances are mismatched, the gains with which the circuit processes v_1 and v_2 will also be mismatched. Moreover, $R_o \neq \infty$. R_o is minimized when R_2, R_3, and R_4 are maximized, R_1 and R_5 are minimized.

$$R_{o(min)} \cong \frac{2\times10^3 \times 98\times10^3}{10^5\times1.001 - (10^5\times0.999)^2/(10^5\times1.001)}$$

$= 490$ kΩ.

2.45



Summing currents at the inverting inputs of the op amps,

$$\frac{v_L - v_1}{9} + \frac{v_2 - v_1}{R_G} + \frac{v_3 - v_1}{10} = 0$$

$$\frac{v_1 - v_2}{R_G} + \frac{v_3 - v_2}{10} + \frac{v_4 - v_2}{10} = 0$$

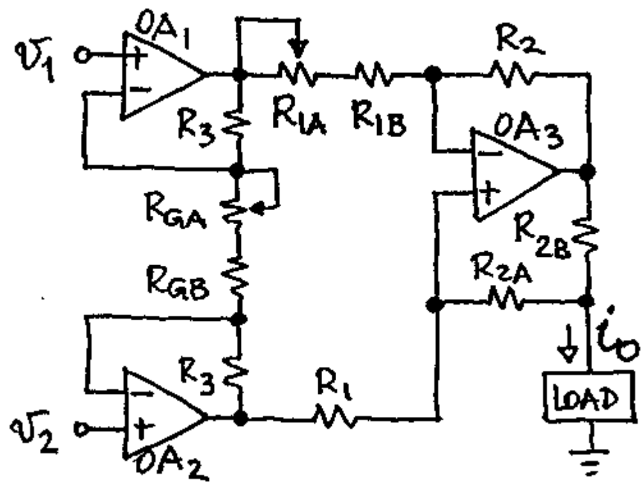
Solving for v_4 gives

$v_4 = \frac{10}{9} v_L + v_2\left(\frac{20}{R_G} + 2\right) - v_1\left(\frac{20}{R_G} + \frac{19}{9}\right)$. KCL:

$i_o = \frac{v_1 - v_L}{9} + \frac{v_4 - v_L}{1}$. Substituting v_4 gives

$i_o = 2\left(1 + \frac{10}{R_G}\right)(v_2 - v_1)$.

2,46 (a) $1/R = A_1/R_1$, $A_1 = 1+2R_3/R_G$. Since $1/R$ must vary over a 100 : 1 range and since $A_1 > 1$, impose $2 \le A_1 \le 200$. Then,



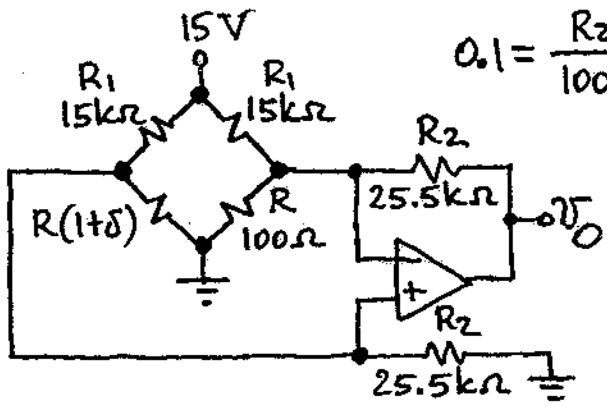
$200 = 1 + 2\frac{R_3}{0+R_{GB}}$,

$2 = 1 + 2\frac{R_3}{100+R_{GB}}$. Solving yields $R_3 = 50.25\ k\Omega$ (use $49.9\ k\Omega$), and $R_{GB} = 0.505\ k\Omega$ (use $499\ \Omega$). When $A_1 = 2$ we want $1/R = 2/R_1 = 1mA/V \Rightarrow R_1 = 2k\Omega$. Use the improved Howland circuit with $R_1 = R_2 = 100\ k\Omega$ and $R_{2B} = 2k\Omega$. Then, $R_{2A} = 100 - 2 = 98\ k\Omega$ (use $97.6\ k\Omega$). Now 4% of $100k\Omega$ is $4k\Omega$. Use $R_{1A} = 10k\Omega$ to be on the safe side, and $R_{1B} = 95.3\ k\Omega$. Summarizing, $R_1 = R_2 = 100k\Omega$, $R_{1A} = 10k\Omega$ pot, $R_{1B} = 95.3k\Omega$, $R_{2A} = 97.6\ k\Omega$, $R_{2B} = 2.00k\Omega$, $R_3 = 49.9\ k\Omega$, $R_{GA} = 100\ k\Omega$ pot, $R_{GB} = 499\ \Omega$.

(b) Let $v_1 = v_2 = 0V$ and adjust R_{1A} as in Fig. 2.9.

2.47 With reference to Fig. 2.34, we want $2R_2R_3/R_1 = 10$ V/mA $= 10$ kΩ. Let $R_1 = R_2 =$ 10.0 kΩ. Then, $R_3 = 10/2 = 5$ kΩ (use 4.99 kΩ, 1%). Moreover, $R_4 = 4.99$ kΩ, 1%.

2.48 (a) Let $R_1 = 15$ kΩ. Then,



$$0.1 = \frac{R_2}{100} 15 \frac{0.00392}{1 + \frac{15{,}000}{100} + \frac{15{,}000}{R_2}}.$$

This yields $R_2 = 170.7\left(151 + \frac{15000}{R_2}\right)$. Starting out with $R_2 = 10$ kΩ and solving by iteration yields $R_2 = 25.8$ kΩ.

(b) $$v_O = \frac{25.5}{0.1} 15 \frac{0.392}{\frac{15}{0.1} + \left(1 + \frac{15}{25.5}\right)\left(1 + 0.392\right)} =$$

9.96 V, which corresponds to a 0.4°C error.

2.49 (a) KCL at the op amp input nodes:

$$\frac{V_{REF} - v_N}{R_1} = \frac{v_N}{R_2} + \frac{v_N - v_O}{R_2} \quad \text{and} \quad \frac{V_{REF} - v_P}{R_1} = \frac{v_P}{R(1+\delta)} + \frac{v_O}{R_2}.$$

Letting $v_N = v_P$ and solving for v_O yields $v_O = (R_2/R)[\delta/(1+\delta)]v_P$. Voltage divider:

$$\frac{v_P}{V_{REF}} = \frac{[R(1+\delta)] /\!/ R_2}{[R(1+\delta)] /\!/ R_2 + R_1} = \frac{1}{1 + \frac{R_1}{[R(1+\delta)] /\!/ R_2}} =$$

$$\frac{1}{1 + R_1 \frac{R(1+\delta) + R_2}{R(1+\delta) R_2}} = \frac{1}{1 + \frac{R_1}{R_2}\left(1 + \frac{R_2}{R}\frac{1}{1+\delta}\right)} =$$

$\frac{1+\delta}{1+\delta\left(1+\frac{R_1}{R_2}\right)+\frac{R_1}{R}}$. Eliminating v_P yields

$$v_O = \frac{R_2}{R} V_{REF} \frac{\delta}{\frac{R_1}{R}+\left(1+\frac{R_1}{R_2}\right)(1+\delta)};$$

$$\lim_{\delta\to 0} v_O = \frac{R_2}{R} V_{REF} \frac{\delta}{1+R_1/R+R_1/R_2}.$$

(b) The output of OA_1 is $v_1 = -\frac{R(1+\delta)}{R_1} V_{REF}$. Superposition: $v_O = -(R_2/R)v_1 - (R_2/R_1)V_{REF}$. Eliminating v_1, $v_O = (R_2/R_1)V_{REF}\,\delta$.

2.50 Impose 1mA through each side of the bridge. Thus, $R_1 = 2.5/2 = 1.25\,k\Omega$. Let $R_2 = 30\,k\Omega$ and $R = 100\,\Omega$, both 1%. Then,

$$0.1 = A\frac{100}{2\times 1250} 2.5\times 0.00392 \Rightarrow A = 255 \text{ V/V}.$$

2.51 (a) Let $i_{RTD} = 1\,mA$, so $R_1 = 15\,k\Omega$. Then,

$$0.1 = \frac{R_2}{15{,}000} 15\times 0.00392 \Rightarrow R_2 = 25.5\,k\Omega.$$

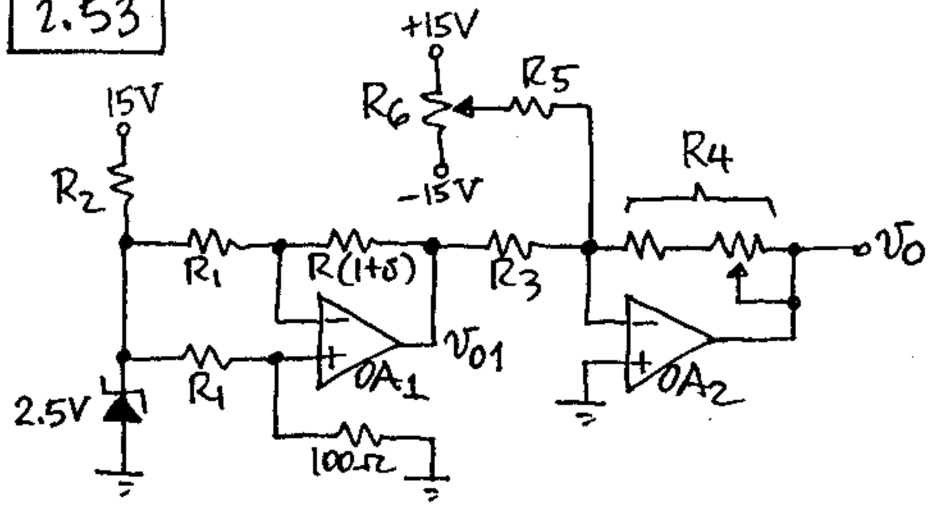
(b) Use the same topology, components, and calibration procedure as in Example 2.13.

2.52 Since $v_N = v_P$, it follows that the two legs of the bridge must conduct identical currents,

$$\frac{V_{REF}-v_O}{R_1+R(1+\delta)} = \frac{V_{REF}}{R_1+R}. \text{ Thus, } v_O = -\frac{R}{R_1+R} V_{REF}\,\delta.$$

The disadvantage is very low sensitivity, thus requiring an additional gain stage.

2.53



Let $R_1 = 2.49\ k\Omega$. Then, $\Delta T = 1°C \Rightarrow \Delta v_{O1} = [100/(100+2490)] \times 2.5 \times 0.00392 = 378.38\ \mu V$.
$\Delta v_O = (R_4/R_3)\Delta v_{O1} = 0.1\ V \Rightarrow R_4/R_3 = 264.3$.
Use $R_3 = 1\ k\Omega$, $R_4 = 237\ k\Omega$ in series with a 50-kΩ pot. Let $R_5 = 3.3\ M\Omega$, $R_6 = 100$-kΩ pot, $R_2 = 3.9\ k\Omega$. To calibrate:
With $T = 0°C$, adjust R_6 for $v_O = 0V$.
With $T = 100°C$, adjust R_4 for $v_O = 10.0V$.

2.54 $v_{N1} = v_{P1} = v_{N2} = v_{P2} = 0\ V$.

$v_{O1} = -[R(1+\delta)/R_1]V_{REF}$. $v_O = -R_2[V_{REF}/R_1 + v_{O1}/R] = -R_2\{V_{REF}/R_1 - [(1+\delta)/R_1]V_{REF}\}$, i.e.
$v_O = (R_2/R_1)V_{REF}\,\delta$.