(3.1)

$\boxed{3.1}$ $$H(s)=\frac{s-10^3}{[s-(-10^3+j10^3)][s-(-10^3-j10^3)]}$$

$$H(s)=\frac{s-10^3}{(s+10^3-j10^3)(s+10^3+j10^3)}=\frac{s-10^3}{s^2+2\times10^3 s+2\times10^6}$$

$$(a)\ A_1=H(s)\times(s+10^3-j10^3)\Big|_{s=-10^3+j10^3}$$

$$=\frac{-10^3+j10^3-10^3}{-10^3+j10^3+10^3+j10^3}=0.5+j1$$

$$=1.118\angle 26.57^\circ .$$

$$h(t)=2.236e^{-10^3t}\cos(10^3t+26.57^\circ)u(t)$$

$$(b)\ H(j10^3)=\frac{j10^3-10^3}{-10^6+2\times10^3 j10^3+2\times10^6}=\frac{j1-1}{10^3(1+2j)}$$

$$=6.32\times10^{-4}\angle 71.57^\circ .$$

$$x_0(t)=6.32\times10^{-4}\cos(10^3t+71.57^\circ).$$

$\boxed{3.2}$ (a) $V_m=V_o/2=V_p$. KCL:

$\dfrac{V_i-V_p}{1/j\omega C}=\dfrac{V_p}{R}+\dfrac{V_p-V_o}{1/j\omega C}$. Eliminating V_p and collecting terms, $H=\dfrac{V_o}{V_i}=j(f/f_0)$, $f_0=1/(4\pi RC)$.

(b) Let $C=10$ nF. Then, $R=1/(4\pi\times 100\times10\times10^{-9})=79.6\ \mathrm{k\Omega}$ (use $80.6\ \mathrm{k\Omega}$). Use $R_1=100\ \mathrm{k\Omega}$.

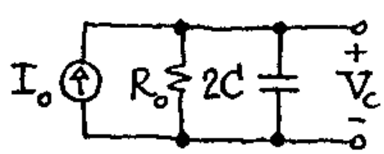
3.3 (a) $V_p = \frac{1}{1+j(f/f_0)} V_i$, $V_n = \frac{j(f/f_0)}{1+j(f/f_0)} V_o$.

$V_n = V_p \Rightarrow V_o = \frac{1}{j(f/f_0)} V_i$, $f_0 = \frac{1}{2\pi R_1 C_1}$.

(b) 20 dB gain at $f = 100$ Hz $\Rightarrow$ $f_0 = 1$ kHz. Let $C_1 = C_2 = 10$ nF. Then, $R_1 = R_2 = 1/(2\pi \times 10^3 \times 10^{-8}) = 15.8$ kΩ.

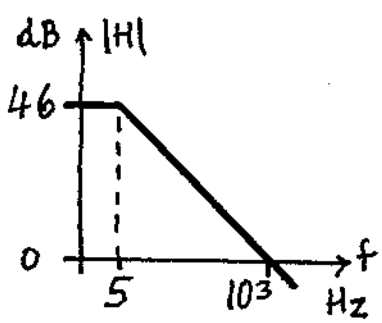
3.4 (a) Let $2C = 10$ nF. Then $R = 1/2\pi f_0 C = 1/(2\pi \times 10^3 \times 5 \times 10^{-9}) = 31.8$ kΩ (use 31.6 kΩ, 1%).

(b) Because of mismatches, $2C$ sees a Norton equivalent with $I_o = V_i/R$ and $R_o = 31.8/[31.8/31.8 - 31.8(1-0.01)/31.8] = 3.180$ MΩ, as per Eq. (2.8). By Ohm's law,



$V_c = \left(R_o \| \frac{1}{s2C}\right) I_o = \frac{R_o/R}{1+jf/f_1} V_i$,

$f_1 = \frac{1}{2\pi R_o 2C} = 5$ Hz. Since



$V_o = \left(1 + \frac{31.8(0.99)}{31.8}\right) V_c \cong 2V_c$,

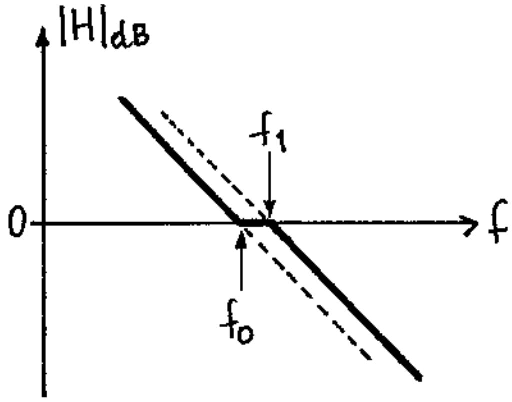
it follows that

$H \cong \frac{2R_o/R}{1+jf/f_1} = \frac{200}{1+jf/5}$.

3.5 (a) Let $f_0 = 1/(2\pi R_2 C_2)$. Then, $1/(2\pi R_1 C_1) = 1/[2\pi R_2 C_2(1-\varepsilon)] \cong (1+\varepsilon)/(2\pi \times R_2 C_2) = f_0(1+\varepsilon) \triangleq f_1$; $V_p = [1/(1+jf/f_1)] V_i$; $V_n = [(jf/f_0)/(1+jf/f_0)] V_o$. Letting $V_n = V_p$ gives

3.3

$$H = \frac{V_o}{V_i} = \frac{1}{jf/f_0} \times \frac{1 + jf/f_0}{1 + jf/f_1}.$$

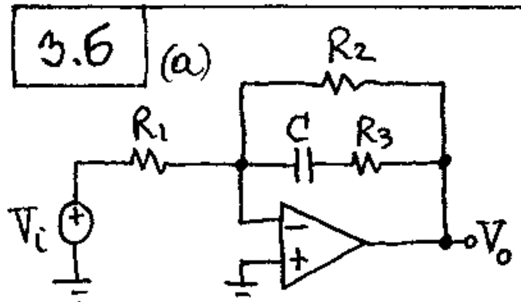


$$|H|_{(f \ll f_0)} \to \frac{1}{f/f_0}$$

$$|H|_{(f \gg f_0)} \to \frac{1}{f/f_1}.$$

(b) Lift R_2, apply a common ac signal to R_1 and R_2, and ajust one of the two resistors until the output is minimized.

3.6 (a)



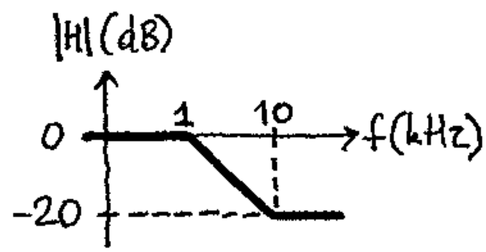
Let $Z_s = 1/sC + R_3 = (1 + sR_3C)/sC$.

Let $\frac{1}{Z_p} = \frac{1}{Z_s} + \frac{1}{R_2}$

Eliminating Z_s gives $\frac{1}{Z_p} = \frac{1 + s(R_2 + R_3)C}{R_2(1 + sR_3C)}$, so

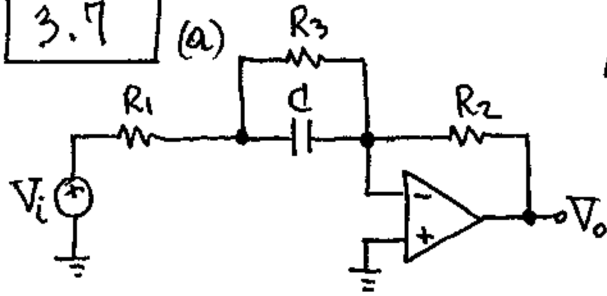
$$H = -\frac{Z_p}{R_1} = -\frac{R_2}{R_1}\frac{1 + jf/f_z}{1 + jf/f_p},\ f_z = \frac{1}{2\pi R_3 C},\ f_p = \frac{1}{2\pi(R_2 + R_3)C}.$$

(b) Let $C = 1$ nF. Then, $R_3 = 1/(2\pi \times 10^4 \times 10^{-9})$ $= 15.9$ kΩ (use 15.8 kΩ, 1%); $R_1 = R_2 = 159 - 15.9$ $= 143$ kΩ, 1%.



3.4

3.7 (a)



Let $Z_p = R_3 \| \frac{1}{sC}$

$= \frac{R_3}{1+sR_3C}$.

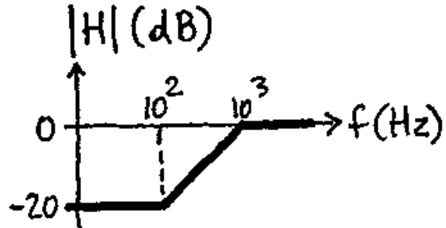
Let $Z_s = R_1 + Z_p$.

Eliminating Z_p gives $Z_s = (R_1+R_3)\frac{1+s(R_1\|R_3)C}{1+sR_3C}$, so

$$H = -\frac{R_2}{Z_s} = -\frac{R_2}{R_1+R_3}\frac{1+jf/f_z}{1+jf/f_p},\ f_z = \frac{1}{2\pi R_3C},\ f_p = \frac{1}{2\pi(R_1\|R_3)C}.$$

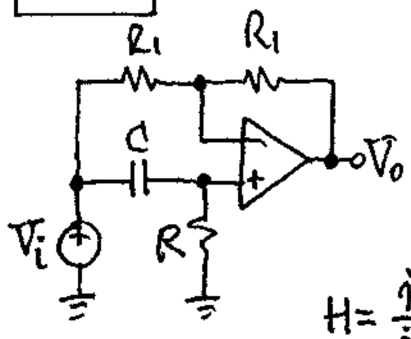
(b) Let $C = 10$ nF. Then, $R_3 = 1/(2\pi \times 100 \times 10^{-8}) =$ 159 kΩ (use 158 kΩ, 1%);

$R_1 \| R_3 = 15.9$ kΩ $\Rightarrow R_1 =$ 17.7 kΩ (use 17.8 kΩ, 1%)



$$H(f\to\infty) = -\frac{R_2}{R_1+R_3}\frac{f_p}{f_z} = -\frac{R_2}{R_1} = -1 \Rightarrow R_2 = 17.8 \text{ k}\Omega.$$

3.8

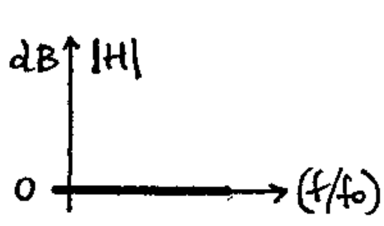


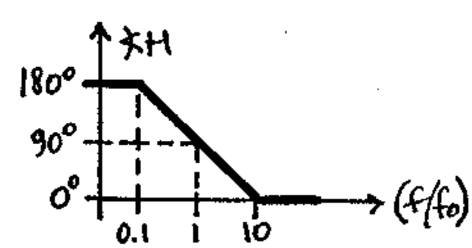
$V_p = \frac{jf/f_0}{1+jf/f_0}$, $f_0 = \frac{1}{2\pi RC}$.

Superposition: $V_o = 2V_p - V_i$.

Eliminating V_p gives

$$H = \frac{jf/f_0 - 1}{jf/f_0 + 1} = 1\ \underline{/180° - 2\tan^{-1}(f/f_0)}.$$

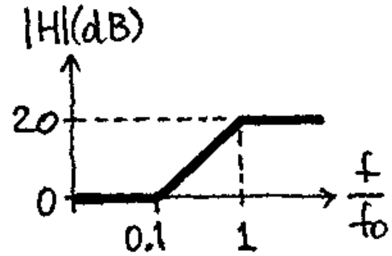


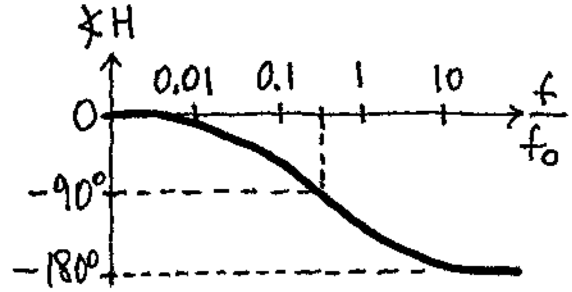


Difference: $\measuredangle H(f/f_0 = 1) = +90°$ instead of $-90°$; disadvantage: C passes high-frequency noise to output.

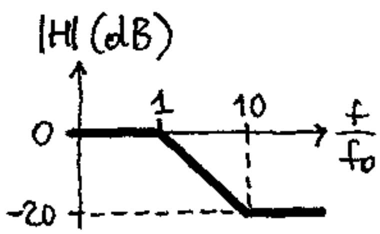
3.5

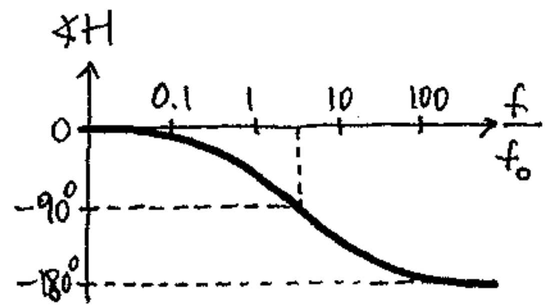
3.9 (a) Superposition: $V_o = -10V_i + 11V_p$, $V_p/V_i = 1/(1+jf/f_0)$, $f_0 = 1/2\pi RC$. Eliminating V_p gives $H = \dfrac{1 - jf/(f_0/10)}{1 + jf/f_0}$, whose Bode plots are:



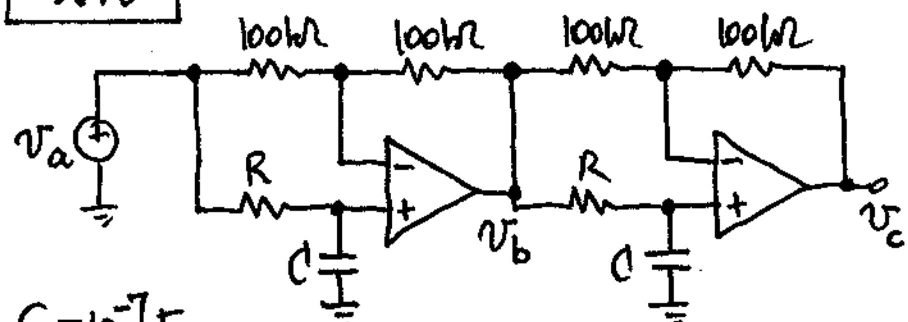


(b) We now have $V_o = -0.1V_i + 1.1V_p$, so $H = \dfrac{1 - jf/10f_0}{1 + jf/f_0}$, whose Bode plots are:





3.10

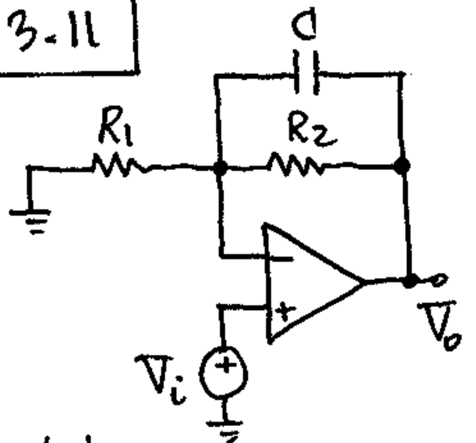


$C = 10^{-7}$ F

$\phi = -2\tan^{-1}(f/f_0) \Rightarrow -120° = -2\tan^{-1}(2\pi 60 R \times 10^{-7})$

$\Rightarrow R = 45.9$ kΩ.

3.11

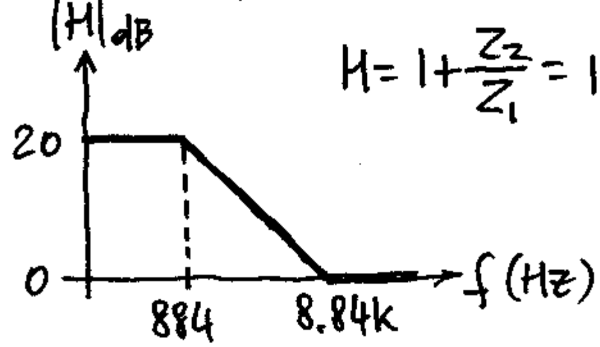


Let $Z_2 = R_2 \| \frac{1}{j\omega C}$, or

$$Z_2 = \frac{R_2}{1+j\omega R_2 C} = \frac{R_2}{1+jf/f_0}$$

$$f_0 = \frac{1}{2\pi R_2 C} = 884.2\,\text{Hz}$$

Let $Z_1 = R_1$. Then,

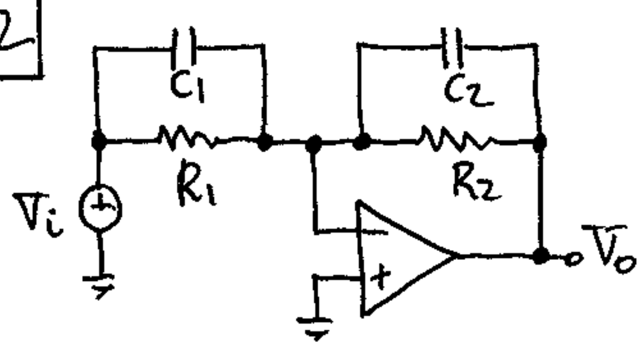


$$H = 1 + \frac{Z_2}{Z_1} = 1 + \frac{18}{2}\,\frac{1}{1+jf/f_0}$$

$$= \frac{1+jf/f_0+9}{1+jf/f_0}$$

$$= 10\,\frac{1+jf/(10f_0)}{1+jf/f_0}.$$

3.12



$$R \| (1/j2\pi f C) = \frac{R}{1+jf/f_0}, \quad f_0 = \frac{1}{2\pi R C}$$

$$H = -\frac{Z_2}{Z_1} = -\frac{R_2}{1+jf/f_2} \times \frac{1+jf/f_1}{R_1} = -\frac{R_2}{R_1}\,\frac{1+jf/f_1}{1+jf/f_2}$$

$f_1 = 1/2\pi R_1 C_1$, $f_2 = 1/2\pi R_2 C_2$, $H_0 = -R_2/R_1$,

$H_\infty = -(R_2/R_1)\times(f_2/f_1) = C_1/C_2$. $H_0 = 20\,\text{dB} \Rightarrow R_2 = 100R_1$.

$H_\infty = 0\,\text{dB} \Rightarrow C_1 = C_2$. $(f_1 f_2)^{1/2} = 10^3 \Rightarrow 20\pi R_1 C_1 = 10^{-3}$.

Let $C_1 = C_2 = 10\,\text{nF}$. Then, $R_1 = 1.591\,\text{k}\Omega$, $R_2 = 159.1\,\text{k}\Omega$.

$\boxed{3.13}$ $V_o = \left(1 + \frac{1}{jwR_2C_2}\right) \frac{1}{1+jwR_1C_1} V_i$, so

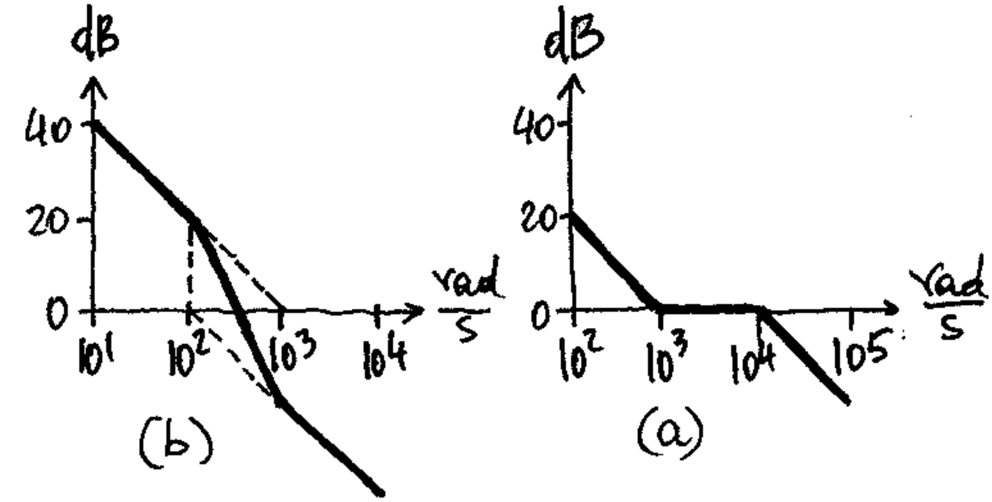
$$H = \frac{1}{jwR_2C_2} \frac{1+jwR_2C_2}{1+jwR_1C_1}$$

(a) $R_1C_1 = 0.1\,ms$, $R_2C_2 = 1\,ms$,

$$H = \frac{1}{jw/10^3} \frac{1+jw/10^3}{1+jw/10^4}$$

(b) $R_1C_1 = 10\,ms$, $R_2C_2 = 1\,ms$,

$$H = \frac{1}{jw/10^3} \frac{1+jw/10^3}{1+jw/10^2}$$



$\boxed{3.14}$ With equal component values, we get

$$H = -\frac{jwRC}{(1+jwRC)^2}$$

(a) $w = 1/RC \Rightarrow wRC = 1 \Rightarrow H = \frac{-j1}{(1+j1)^2} =$

$-j1/(1-1+j2) = -0.5 = 0.5\angle{-180°}$. Thus,

$v_o(t) = 0.5 \cos(t/RC - 180°)$ V.

(b) $\omega = 1/2RC \Rightarrow j\omega RC = j0.5 \Rightarrow$

$$H = \frac{-j0.5}{(1+j0.5)^2} = 0.4\underline{/-90^\circ - 2\tan^{-1}0.5} = 0.4\underline{/-143^\circ}$$

$\Rightarrow v_o(t) = 0.4\cos(t/2RC - 143^\circ)$ V.

(c) $\omega = 2/RC \Rightarrow j\omega RC = 2 \Rightarrow H = 0.4\underline{/+143^\circ}$

$\Rightarrow v_o(t) = 0.4\cos(2t/RC + 143^\circ)$ V.

$\boxed{3.15}$ (a) Let V_1 and V_2 be the outputs of OA_1 and OA_2. Applying a test voltage V at the input gives $V_1 = V$, and $V_2 = (-R_2/R_1)V$. The current out of the test source is $I = (V - V_2)/(1/sC) = sC(1 + R_2/R_1)V$. Then, $Z_{eq} = V/I = 1/sC_{eq}$, $C_{eq} = (1 + R_2/R_1)C$.

(b) $C = 0.1\,\mu F$, $R_1 = 1\,k\Omega$, $R_2 = 1$-$M\Omega$ pot connected as a variable resistance from 0 to $1\,M\Omega$.

$\boxed{3.16}$ (a) Applying a test voltage V at the input gives $V_{n2} = V_{p2} = V_{p1} = V_{n1} = V$; moreover, $V_{O2} = (1 + R_3/R_4)V$, $(V_{O1} - V)/R_2 = (V - V_{O2})/(1/sC)$, and $I = (V - V_{O1})/R_1$, where I is the current out of the test source. Eliminating V_{O1} and V_{O2} gives $I = sC_{eq}V$, $C_{eq} = (R_2R_3/R_1R_4)C$.

(b) $C = 1\,nF$, $R_1 = R_4 = 1\,k\Omega$, $R_2 = R_3 = 1\,M\Omega$. C_{eq} could be used to create very long time constants without using excessively large R's.

3.17 Let $Z_2 = R_2 // (1/j\omega C_2) = R_2/(1+j\omega R_2 C_2)$, and $Z_3 = R_3//(1/j\omega C_3) = R_3/(1+j\omega R_3 C_3)$. Then, $H = 1 + (Z_2+Z_3)/R_1$, that is,

$$H = 1 + \frac{1}{R_1}\frac{R_2(1+j\omega R_3 C_3) + R_3(1+j\omega R_2 C_2)}{(1+j\omega R_2 C_2)\times(1+j\omega R_3 C_3)} =$$

$$1 + \frac{1}{R_1}\frac{(R_2+R_3) + j\omega R_2 R_3 (C_2+C_3)}{(1+j\omega R_2 C_2)\times(1+j\omega R_3 C_3)} =$$

$$1 + \frac{1}{R_1}\frac{1+j\omega (R_2//R_3)(C_2+C_3)}{(1+j\omega R_2 C_2)\times(1+j\omega R_3 C_3} =$$

$$1 + \frac{R_2+R_3}{R_1}\frac{1+j(f/f_1)}{[1+j(f/f_2)][1+j(f/f_3)}\,,$$

$$f_1 = \frac{1}{2\pi (R_2//R_3)(C_2+C_3)},\quad f_2 = \frac{1}{2\pi R_2 C_2},\quad f_3 = \frac{1}{2\pi R_3 C_3}.$$

3.18 (a) Let $Z_2 = R_2 + 1/j\omega C_2$, that is,

$$Z_2 = R_2 \frac{1+j\omega R_2 C_2}{j\omega R_2 C_2}.$$ Let $Z_F = R_3 // Z_2$, that is,

$$Z_F = \frac{R_3 \times R_2 \dfrac{1+j\omega R_2 C_2}{j\omega R_2 C_2}}{R_3 + R_2 \dfrac{1+j\omega R_2 C_2}{j\omega R_2 C_2}} =$$

$$R_3 \frac{R_2(1+j\omega R_2 C_2)}{j\omega R_2 R_3 C_2 + R_2(1+j\omega R_2 C_2)}.$$ Then,

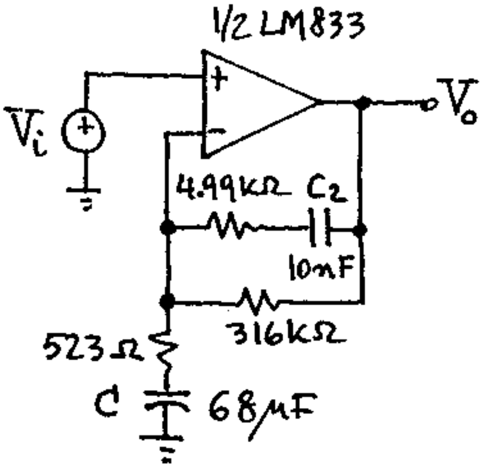
$$H = 1 + \frac{Z_F}{R_1} = 1 + \frac{R_3}{R_1}\frac{1+j\omega R_2 C_2}{1+j\omega (R_2+R_3) C_2} =$$

$$1 + \frac{R_3}{R_1}\frac{1+j(f/f_1)}{1+j(f/f_2)},\quad f_1 = \frac{1}{2\pi R_2 C_2},\quad f_2 = \frac{1}{2\pi (R_2+R_3) C_2}.$$

(3.10)

(b) Let $C_2 = 10nF$. Then,

$$R_2 = \frac{1}{2\pi \times 3{,}183 \times 10^{-8}} = 5k\Omega.$$



$$R_3 = \frac{1}{2\pi \times 50 \times 10^{-8}} - 5k = 313k\Omega.$$

For $f = 1\,kHz$,

$$|H| \cong \frac{R_3}{R_1}\sqrt{\frac{1+(10^3/3183)^2}{1+(10^3/50)^2}} = \frac{R_3}{R_1} \times 5.23 \times 10^{-2}.$$

$$R_1 = \frac{5.23 \times 10^{-2}}{10^{30/20}} \times 313 = 518\Omega \text{ (use } 523\Omega).$$

$$C \cong 10\frac{1}{2\pi \times 50 \times 518} = 61\mu F \text{ (use } 68\mu F).$$

3.19 Using $R_1 = 10k\Omega$, $R_2 = 100k\Omega$, and $R_3 = 1M\Omega$, we obtain $C_2 = (2 + 100/10)^{1/2}/(20\pi \times 10^5 f_0) = (0.551 \times 10^{-6})/f_0$. Moreover, $C_1 = 10C_2$. Using the above equations repeatedly, we obtain:

32Hz : $C_2 = 18nF$, $C_1 = 180nF$;

64Hz : $C_2 = 9.1nF$, $C_1 = 91nF$;

125Hz : $C_2 = 4.7nF$, $C_1 = 47nF$;

250Hz : $C_2 = 2.2nF$, $C_1 = 22nF$;

500 Hz : $C_2 = 1nF$, $C_1 = 10nF$;

1KHz : $C_2 = 560pF$, $C_1 = 5.6nF$;

2KHz : $C_2 = 270pF$, $C_1 = 2.7nF$;

(3.11)

4KHz: $C_2 = 130\,pF$, $C_1 = 1.3\,nF$;
8KHz: $C_2 = 68\,pF$, $C_1 = 680\,pF$
16KHz: $C_2 = 33\,pF$, $C_1 = 330\,pF$.

3.20 (a) $H = H_0 \dfrac{j\omega/\omega_L}{1 - \omega^2/\omega_L\omega_H + j\omega(1/\omega_L + 1/\omega_H)} =$

$H_{0BP} \dfrac{(j\omega/\omega_0)/Q}{1-(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}$, where $\omega_0^2 = \omega_L \omega_H$

$\dfrac{1}{Q\omega_0} = \dfrac{1}{\omega_L} + \dfrac{1}{\omega_H} = \dfrac{\omega_L+\omega_H}{\omega_L\omega_H} = \dfrac{\omega_L+\omega_H}{\omega_0^2}$. Thus, $Q = \dfrac{\omega_0}{\omega_L+\omega_H}$.

(b) Let $\omega_H = n\omega_L$. Then, $Q = \sqrt{n}/(n+1)$. This reaches its maximum when $n=1$, where $Q = 0.5$. Thus, no matter how we choose ω_L and ω_H, we always have $Q \le 0.5$.

3.21 (a) Low-pass: $\measuredangle H_{LP(\omega/\omega_0 \ll 1)} \cong \measuredangle 1 = 0°$;

$\measuredangle H_{LP(\omega/\omega_0 = 1)} = \measuredangle \dfrac{1}{0 + (j/Q)1} = -90°$;

$\measuredangle H_{LP(\omega/\omega_0 \gg 1)} \cong \measuredangle 1/-(\omega/\omega_0)^2 = -180°$.

The phase at other frequencies is found as

$\measuredangle H_{LP} = -\tan^{-1} \dfrac{(\omega/\omega_0)/Q}{1-(\omega/\omega_0)^2}$. For instance,

$\measuredangle H_{LP(\omega/\omega_0 = 0.5)} = -\tan^{-1} \dfrac{0.5/Q}{1-0.25} = -\tan^{-1} \dfrac{2/3}{Q}$,

3.12

indicating that the higher the value of Q, the closer the phase to $0°$ at $(\omega/\omega_0) = 0.5$. This is readily generalized by saying that the higher the value of Q, the narrower the transition region from $0°$ to $-180°$ (see Fig. 1).

(b) $\measuredangle H_{HP} = \measuredangle[-(\omega/\omega_0)^2] + \measuredangle H_{LP} = 180° + \measuredangle H_{LP}$, indicating that the plot of $\measuredangle H_{HP}$ can be obtained by shifting that of $\measuredangle H_{LP}$ upward by $180°$ (see Fig. 2).

(c) $\measuredangle H_{BP} = \measuredangle[(j/Q)(\omega/\omega_0)] + \measuredangle H_{LP} = 90° + \measuredangle H_{LP}$, indicating that the plot of $\measuredangle H_{BP}$ can be obtained by shifting that of $\measuredangle H_{LP}$ upward by $90°$ (see Fig. 3).

(d) By inspection, $\measuredangle H_N = \measuredangle H_{LP}$ for $\omega/\omega_0 < 1$, $\measuredangle H_N = \measuredangle H_{HP}$ for $\omega/\omega_0 > 1$. Thus, the plot of $\measuredangle H_N$ is as in Fig. 4. Note the phase discontinuity at $\omega/\omega_0 = 1$.

3.13

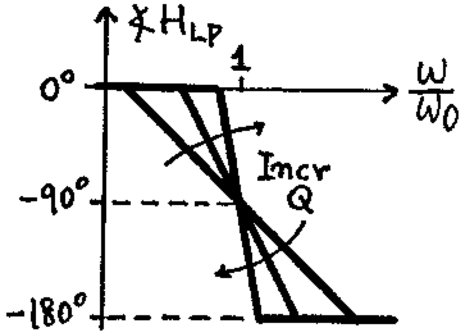


Fig. 1

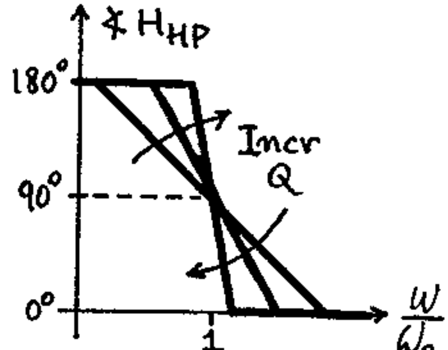


Fig. 2

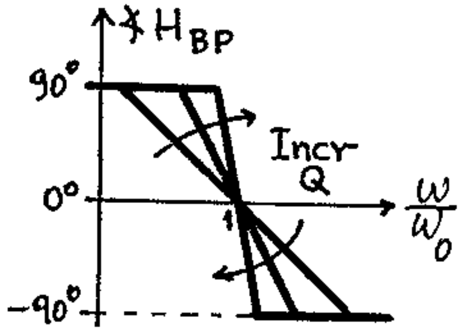


Fig. 3

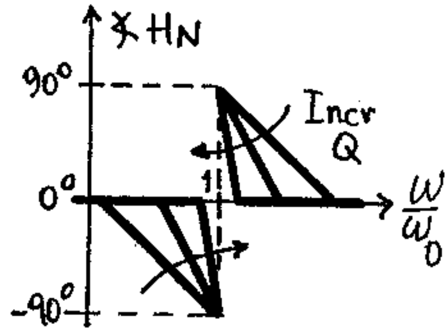


Fig. 4

$\boxed{3.22}$ (a) With $R_A=R_B$, Eq. (3.60a) gives $H_{0LP}=K=2$ V/V. With $R_2/R_1=C_1/C_2=Q$, Eq. (3.60b) gives $\omega_0=1/R_1C_1$, so the design equation is $R_1=1/\omega_0C_1$.

(b) Let $C_1=10$ nF, so that $C_2=C_1/Q=$ 2 nF. Then, $R_1=1/(2\pi 10^3\times 10^{-8})=15.9$ kΩ (use 15.8 kΩ, 1%), and $R_2=QR_1=78.7$ kΩ, 1%. Use also $R_A=R_B=15.8$ kΩ, 1%.

$\boxed{3.23}$ (a) With $C_1=C_2=C$, Eq (3.60b) gives $R_1=1/\omega_0^2C^2R_2$, and Eq. (3.60c) gives $1/Q=(2-K)\times\sqrt{R_1/R_2}+\sqrt{R_2/R_1}=(2-K)/\omega_0CR_2+\omega_0CR_2$, or

(3.14)

$\omega_0 C R_2^2 - (1/Q)R_2 + (2-K)/\omega_0 C = 0$, whose phisically acceptable solution, for $K = H_{0LP} > 2$, is

$$R_2 = \left[1 + \sqrt{1 + 4Q^2(H_{0LP} - 2)}\right]/2\omega_0 Q C.$$

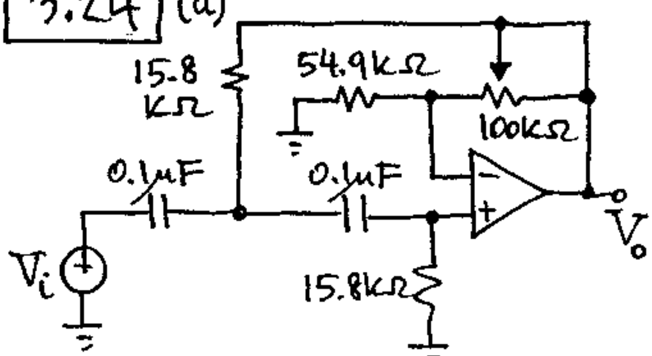
(b) Let $C_1 = C_2 = 10$ nF. Then,

$$R_2 = \frac{1 + \sqrt{1 + 4\times 5^2(10-2)}}{2\times 2\pi\times 10^3 \times 5\times 10^{-8}} = 46.6\ \text{k}\Omega\ (\text{Use } 46.4\ \text{k}\Omega)$$

$$R_1 = \frac{1}{(2\pi 10^3)^2 \times 46.6\times 10^3 \times (10^{-8})^2} = 5.43\ \text{k}\Omega\ (\text{Use}$$

5.49 kΩ). Moreover, for $K = 1 + R_B/R_A = 10$, Use $R_A = 10.0$ kΩ and $R_B = 90.9$ kΩ, all 1%.

$\boxed{3.24}$ (a)



Let $C = 0.1\mu$F. Then,

$$R = \frac{1}{2\pi\times 100\times 10^{-7}}$$

$= 15.9$ kΩ.

Use 15.8kΩ.

$R_B/R_A = K - 1 = 3 - 1/Q - 1 = 2 - 1/Q$.

$(Q = 0.5) \Rightarrow R_B/R_A = 2 - 1/0.2 = 0$

$(Q = 5) \Rightarrow R_B/R_A = 2 - 1/5 = 1.8$. Thus,

$R_A = 100/1.8 = 55.5$ kΩ (use 54.9kΩ).

(b) The DC component is blocked out, and since $\omega/\omega_0 = 60/100 = 0.6$, the ac component is magnified by

$$H = \frac{-(3-1/Q)0.6^2}{1 - 0.6^2 + j0.6/Q} = \frac{0.5625(1/Q - 3)}{1 + j0.9375/Q}.$$

3.15

For $Q=0.5$, $H=\dfrac{-0.5625}{1+j1.875}$, so that $|H|=$
$\dfrac{0.5625}{\sqrt{1+1.875^2}} = 0.265$, $\measuredangle H = 180° - \tan^{-1} 1.875 =$
$118°$. Thus, $V_{om} = 0.265 \times 5 \times \sqrt{2} = 1.874$ V, so that
$v_o(t) = 1.874 \cos(2\pi 60 t + 118°)$ V.

Likewise, for $Q=5$, it is found that
$v_o(t) = 10.95 \cos(2\pi 60 t + 169°)$ V.

3.25 (a) With $C_1=C_2=C$, Eq. (3.65a) gives $R_2 = 1/\omega_0^2 C^2 R_1$, and Eq. (3.65b) gives $1/Q =$
$= (1-K)\sqrt{R_2/R_1} + 2\sqrt{R_1/R_2} = (1-K)/\omega_0 R_1 C + 2\omega_0 R_1 C$,
or $(2\omega_0 C)R_1^2 - (1/Q)R_1 + (1-K)/\omega_0 C = 0$. Solving and letting $K \to H_{0HP}$ gives, for $K>1$, $R_1 =$
$[1+\sqrt{1+8Q^2(H_{0HP}-1)}]/4\omega_0 QC$.

(b) Let $C_1=C_2=10$ nF. Then,

$R_1 = \dfrac{1+\sqrt{1+8\times 0.5(10-1)}}{4\times 2\pi 10^3 \times 10^{-8}/\sqrt{2}} = 39.9$ kΩ (use 40.2 kΩ)

$R_2 = \dfrac{1}{(2\pi 10^3 \times 10^{-8})^2 \times 39.9 \times 10^3} = 6.35$ kΩ (use 6.34 kΩ). Moreover, for $H_{0HP} = 10$ V/V use $R_A = 10.0$ kΩ and $R_B = 90.9$ kΩ, all 1%.

3.26 With $C_1=C_2=C$ and $K=2$, Eqs. (3.66) give $H_{0BP} = 2/(1+2R_1/R_2 - R_1/R_3)$.
$\dfrac{\omega_0}{Q} = \dfrac{1}{C}\left(\dfrac{1}{R_1} - \dfrac{2}{R_2} - \dfrac{1}{R_3}\right)$ and $\omega_0^2 = \dfrac{1}{R_2 C^2}\left(\dfrac{1}{R_1} + \dfrac{1}{R_3}\right)$

(3.16) To simplify our calculations, let us first design for $\omega_0 = 1$ rad/s with $C = 1$ F and $R_1 = 1\ \Omega$; then we suitably rescale all components. Our equations are now $1/Q = (1 - 2/R_2 - 1/R_3)$, $1 = (1 + 1/R_3)/R_2$. Letting $1/R_2 \to G_2$ and $1/R_3 \to G_3$ gives $1/5 = 1 + 2G_2 - G_3$, $1 = G_2(1 + G_3)$. Eliminating G_2 gives $G_3^2 + 0.2G_3 - 2.8 = 0$, $G_2 = 1/(1 + G_3)$.

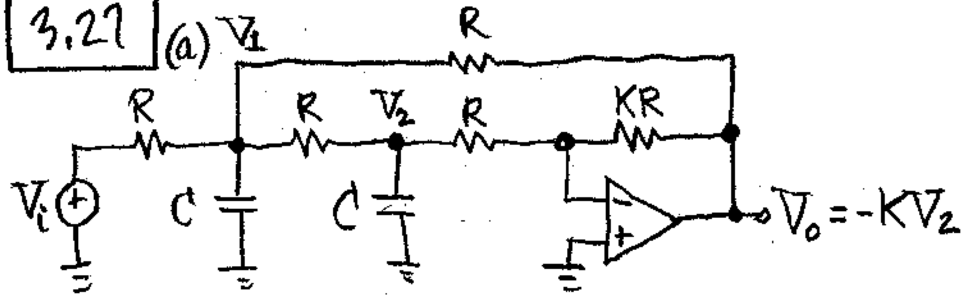
$$G_3 = \frac{-0.2 + \sqrt{0.2^2 + 4 \times 2.8}}{2} = 1.5763 \Rightarrow R_3 = 0.6344\ \Omega,$$

$R_2 = 2.5763\ \Omega$, $H_{0BP} = 10$ V/V.

Use $C = 10$ nF, indicating that all resistances must be increased by $(1\text{ F})/(10\text{ nF}) = 10^8$, and then decreased by $\omega_0/(1\text{ rad/s}) = 2\pi 10^3$, for a total of $10^8/2\pi 10^3 = 10^5/2\pi$. Thus, $R_1 = 1 \times 10^5/2\pi = 15.9$ kΩ (use 15.8 kΩ), $R_2 = 41.0$ kΩ (use 41.2 kΩ), and $R_3 = 10.1$ kΩ (10.0 kΩ). Finally, to ensure $H_{0BP} = 0$ dB, replace R_1 with a voltage divider as in Eq. (3.63) to get $R_{1A} = 158$ kΩ and $R_{1B} = 17.8$ kΩ, all 1%. The circuit checks with PSpice.

3.17

3.27 (a)



Summing currents @ V_1 and V_2:

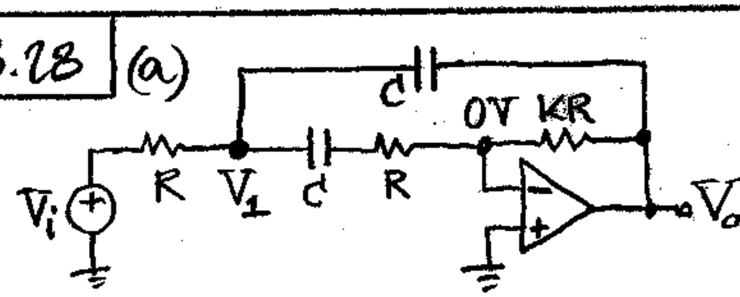
$$\frac{V_i - V_1}{R} - sCV_1 + \frac{V_2 - V_1}{R} + \frac{V_o - V_1}{R} = 0$$

$$\frac{V_1 - V_2}{R} - sCV_2 - \frac{V_2}{R} = 0$$. Eliminating V_1 and V_2 gives

$H = V_o/V_i = H_{0LP} \times H_{LP}$, where $H_{0LP} = -K$, $\omega_0 = \sqrt{K+5}/RC$, and $Q = \sqrt{K+5}/5$.

(b) $K = 25Q^2 - 5 = 620$. Let $C = 0.5\ \mu F$; then $R = \sqrt{620+5}/(0.5 \times 10^{-6} \times 2\pi \times 2 \times 10^3) = 3.98$ kΩ, and $KR = 2.47$ MΩ (use 2.49 MΩ, 1%). For 0-dB dc gain, replace R_1 with a voltage divider as per Eq. (3.63). This gives $R_{1A} = 2.49$ MΩ and $R_{1B} = 4.02$ kΩ.

3.28 (a)



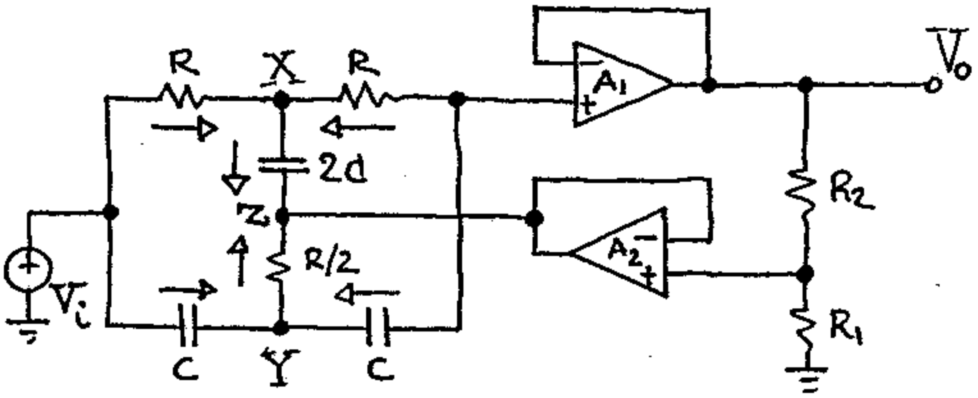
$$V_o = -\frac{KR}{R + 1/sC} V_1 ; \quad \frac{V_i - V_1}{R} + \frac{V_o - V_1}{1/sC} - \frac{V_1}{R + 1/sC} = 0$$

Eliminating V_1 gives $H = V_o/V_i = H_{0BP} \times H_{BP}$, $\omega_0 = 1/(RC\sqrt{K+1})$, $Q = \sqrt{K+1}/3$, $H_{0BP} = -K/3$.

3.18

(b) $K = 9Q^2 - 1 = 899$; let $C = 3.3\,nF$; then $R = 1/(\sqrt{900} \times 2\pi \times 10^3 \times 3.3 \times 10^{-9}) = 1.607\ k\Omega$, and $KR = 1.445\ M\Omega$ (use $1.43\ M\Omega$, 1%). To lower the resonance gain from $-899/3$ V/V to -1 V/V, replace R_1 with a voltage divider consisting of $R_{1A} = 1.43\ M\Omega$ and $R_{1B} = 1.43\ k\Omega$.

3.29 (a)



$V_{P1} = V_{M1} = V_o$. $V_Z = V_{P2} = KV_o$, $K = R_1/(R_1+R_2)$.

KCL at X:

$$(V_i - V_x)/R + (V_o - V_x)/R = j\omega 2C(V_x - KV_o).$$

Solving for V_x:

$$V_x = \frac{V_i + (1 + 2j\omega KRC)V_o}{2(1 + j\omega RC)}.$$

KCL at Y:

$$j\omega C(V_i - V_y) + j\omega C(V_o - V_Y) = (V_y - kV_o)/(R/2).$$

Solving for V_y:

$$V_y = \frac{j\omega RCV_i + (2k + j\omega RC)V_o}{2(1 + j\omega RC)}.$$

3.19

Superposition principle at A_1's noninverting input:

$$V_o = V_{P1} = \frac{1}{1+j\omega RC}V_x + \frac{j\omega RC}{1+j\omega RC}V_y.$$

Substituting V_x and V_y and collecting:

$$V_i(1-\omega^2R^2C^2) = [1-\omega^2R^2C^2 + j4(1-k)\omega RC]V_o.$$

Letting $\omega^2R^2C^2 = (\omega/\omega_0)^2$, so that $\omega_0 = 1/RC$ and $Q = 1/[4(1-K) = (1+R_1/R_2)/4$, we have

$$H = \frac{V_o}{V_i} = \frac{1-(f/f_0)^2}{1-(f/f_0)^2 + (j/Q)(f/f_0)} = H_N.$$

(b) Let $C = 0.1\,\mu F$; then $R = 1/(2\pi 60\times 10^{-7}) =$ 26.5 kΩ (use 26.7 kΩ, 1%); $25 = (1+R_1/R_2)/4$ $\Rightarrow R_1/R_2 = 99$. Pick $R_1 = 100$ kΩ, $R_2 = 1.00$ kΩ.

3.30 With $R_1 = R_2 = R_3 = R$, Eq. (3.74) gives $H_{0LP} = -1$ V/V, $\omega_0 = 1/(R\sqrt{C_1C_2}$, $Q = \sqrt{C_1/C_2}/3$. Pick C_2; then $C_1 = 9Q^2C_2$, $R = 1/3\omega_0 Q C_2$.

3.31 (a) $V_o = -KV_i - K(-2H_{BP}V_i) = -K(1-2H_{BP}) \times V_i = -KH_{AP}V_i$.

(b) $C_1 = C_2 = 10$ nF, $R_2 = 158$ kΩ, $R_{1A} = 40.2$ kΩ, $R_{1B} = 3.32$ kΩ, $R_3 = R_4 = 10.0$ kΩ, $R_5 = 100$ kΩ, all 1%.

3.32 $V_n = V_p = V_i/3$. Let V_1 be the voltage at the left plates of the capacitors. KVL:

$$V_1 = \frac{V_i}{3} + \frac{1}{sC}\,\frac{V_i/3 - V_o}{R} \; ; \text{ KCL:}$$

$$\frac{V_i - V_1}{0.5R} + sC(V_i/3 - V_1) + sC(V_o - V_1) = 0.$$

Eliminating V_1, collecting, and letting $s \to j\omega$,

$$\frac{V_o}{V_i} = \frac{1}{3}\,\frac{1-\omega^2R^2C^2/2 + j\omega RC}{1-\omega^2R^2C^2/2 + j\omega RC} = \frac{1}{3}H_{AP},$$

$$\omega_0 = \frac{\sqrt{2}}{RC}, \quad Q = \frac{1}{\sqrt{2}}.$$

3.33 (a) Denoting the output of OA_1 as V_1, we have $V_o = H_{0BP}H_{BP}V_1 = H_{0BP}H_{BP}[(-R_5/R_3)V_i - (R_5/R_4)V_o]$, where $H_{0BP} = -2Q^2R_{1B}/(R_{1A}+R_{1B})$, and $Q = 0.5\sqrt{R_2/(R_{1A}\|R_{1B})}$. Collecting gives

$$\frac{V_o}{V_i} = H_{0BP(comp)} \times \frac{(j\omega/\omega_0)/Q_{comp}}{1-(\omega/\omega_0)^2 + (j\omega/\omega_0)/Q_{comp}},$$

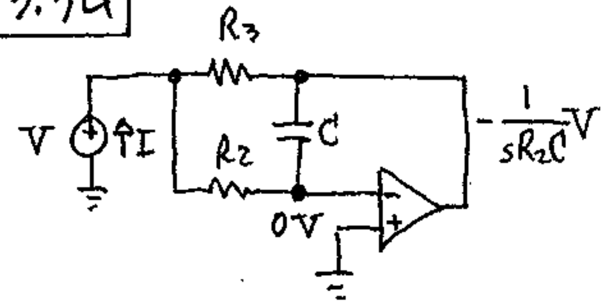
$$\omega_0 = \frac{1}{[R_2(R_{1A}\|R_{1B})]^{1/2}C}, \quad Q_{comp} = \frac{Q}{1-(R_5/R_4)|H_{0BP}|}$$

$$H_{0BP(comp)} = \frac{R_5}{R_3}\,\frac{Q_{comp}}{Q}\,|H_{0BP}|.$$

(b) For simplicity, design the band-pass stage for $|H_{0BP}| = 1$ V/V. A component set is $C = 10$ nF, $R_2 = 88.7$ kΩ, $R_{1A} = 44.2$ kΩ, $R_{1B} = 221$ Ω. To achieve $60 = 10/(1-R_5/R_4)$ and $2 = (R_5/R_3) \times 60/10$ use $R_5 = 100$ kΩ, $R_4 = 121$ kΩ, $R_3 = 301$ kΩ.

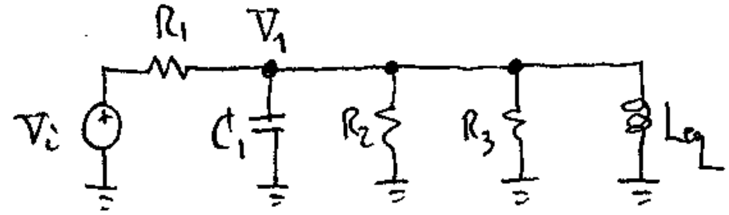
(3.21)

3.34



$$I = \frac{V}{R_2} + \frac{V - (-V/sR_2C)}{R_3} = V\left(\frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{sR_2R_3C}\right);$$

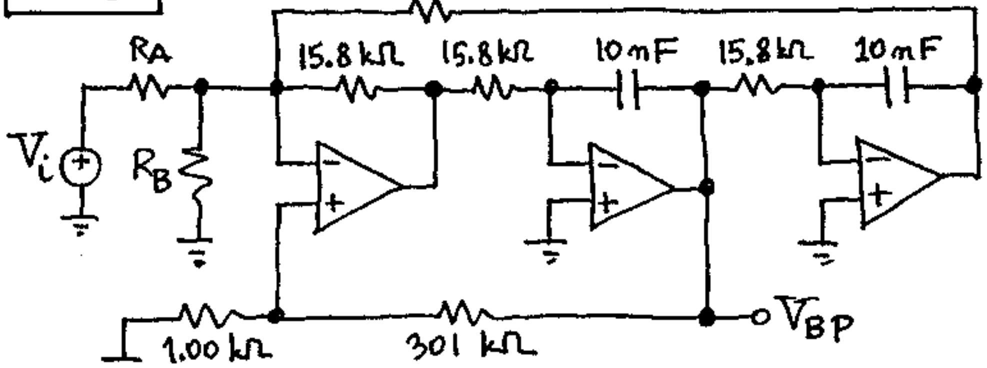
$$Z_{eq} = \frac{V}{I} = R_2 \| R_3 \| sL_{eq}, \quad L_{eq} = R_2R_3C .$$



$$\left.\begin{array}{l} \omega \to 0 \Rightarrow V_1 \to 0 \text{ because of } L_{eq} \\ \omega \to \infty \Rightarrow V_1 \to 0 \text{ because of } C_1 \end{array}\right\} \Rightarrow V_1 = \text{band-pass.}$$

The op amp integrates V_1, indicating that the pole at the origin due to integration cancels out the zero at the origin due to the band-pass response V_1. Consequently, the op amp output is a low-pass response.

3.35



(3.22) Impose $R_A // R_B = \frac{R_A R_B}{R_A + R_B} = 15.8\ k\Omega$ and $\frac{R_B}{R_A + R_B} = \frac{1}{Q} = \frac{1}{100}$. Solving, we obtain $R_A = 1.58\ M\Omega$ and $R_B = 15.8\ k\Omega$.

$\boxed{3.36}$ (a) By the superposition principle, $V_{HP} = KV_i + KV_{BP} - V_{LP}$, where $K = 2\frac{R_1 // R_2}{R_1 // R_2 + R_2} = \frac{2R_1}{2R_1 + R_2}$. Expanding,

$$V_{HP} = KV_i - K\frac{1}{jW/W_0}V_{HP} + \frac{1}{jW/W_0}V_{HP}.$$

Expanding and collecting,

$$[1 - (\omega/\omega_0)^2 + jK(\omega/\omega_0)]V_{HP} = -K(\omega/\omega_0)^2 V_i.$$

Letting $Q = 1/K = 1 + (1/2)R_2/R_1$, we have $\frac{V_{HP}}{V_i} = \frac{1}{Q}H_{HP}$. Likewise, $\frac{V_{BP}}{V_i} = -H_{BP}$, and $\frac{V_{LP}}{V_i} = \frac{1}{Q}H_{LP}$, $\omega_0 = 1/RC$.

(b) $f_0 = (594 \times 606)^{1/2} = 600\text{Hz}$. $Q = 600/(606 - 594) = 50$. Let $C = 10\text{nF}$. Then, $R = 1/(2\pi \times 600 \times 10^{-8}) = 26.5\ k\Omega$ (use $26.7k\Omega$). $R_2/R_1 = 2(Q-1) = 98$. Use $R_1 = 1.02\ k\Omega$, $R_2 = R_3 = 100\ k\Omega$, all 1%.

(c) $H_{0LP} = 1/Q = 1/50 = 0.02\ V/V$.

3.23

$\boxed{3.37}$ (a) $V_{N1} = V_{P1} = V_{LP}$. Integrator:

$$V_{LP} = -\frac{1}{j\omega n RC} V_{BP} \Rightarrow V_{BP} = -j\omega n RC V_{LP}.$$

KCL:

$$\frac{V_i - V_{LP}}{R} = \frac{V_{LP}}{mR} + j\omega C (V_{LP} - V_{BP})$$

Eliminating V_{BP} and collecting,

$$\frac{V_{LP}}{V_i} = \frac{m}{m+1} \frac{1}{1 - \omega^2 R^2 C^2 \frac{mn}{m+1} + j\omega RC \frac{m}{m+1}}.$$

Letting $\omega^2 R^2 C^2 mn/(m+1) = (\omega/\omega_0)^2$, $\omega RC m/(m+1) = (\omega/\omega_0)/Q$, and $H_{0LP} = m/(m+1)$ yields

$H = H_{0LP} H_{LP}$, $\omega_0 = \dfrac{1}{\sqrt{mn/(m+1)}\, RC}$ and $Q = \sqrt{n(m+1)/m}$.

$$\frac{V_{BP}}{V_i} = -j\omega n RC \frac{V_{LP}}{V_i} = -\frac{j\omega n RC\, m/(m+1)}{1 - (\omega/\omega_0)^2 + (j\omega/\omega_0)/Q}.$$

Letting $\omega n RC m/(m+1) = H_{0BP} (\omega/\omega_0)/Q$ yields $H_{0BP} = -n$.

(b) Let $m = \infty$ (mR absent). Then, $Q = 10 \Rightarrow n = 100$. Let $C = 1\,nF$. Then, $R = 7.957\,k\Omega$ (use $8.06\,k\Omega$) and $nR = 806\,k\Omega$. $H_{0BP} = 100\,V/V$.

(c) $H_{0BP} \propto Q^2$, increases quadratically with Q.

3.24

$\boxed{3.38}$ (a) $V_N = \alpha V_{LP} + \beta V_{HP}$ implies

$$\frac{V_N}{V_i} = \alpha \frac{V_{LP}}{V_i} + \beta \frac{V_{HP}}{V_i} = \alpha H_{0LP} H_{LP} + \beta H_{0HP} H_{HP}$$

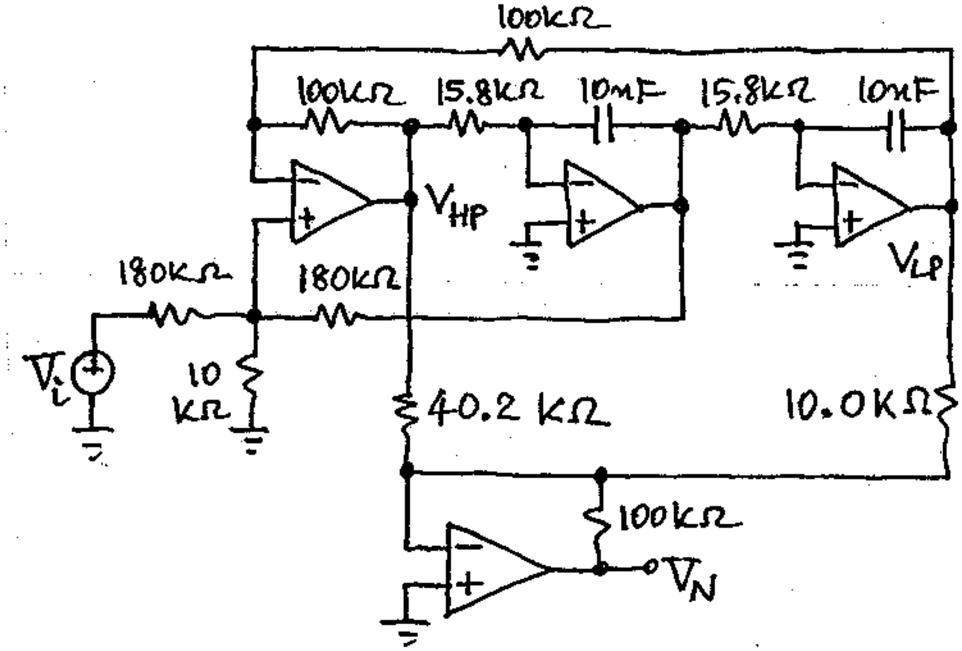
$$H_N = \frac{\alpha H_{0LP} - \beta H_{0HP} (f/f_0)^2}{1-(f/f_0)^2 + (j/Q)(f/f_0)}$$

$$H_N = \alpha H_{0LP} \frac{1-(f/f_z)^2}{1-(f/f_0)^2+(j/Q)(f/f_0)},$$

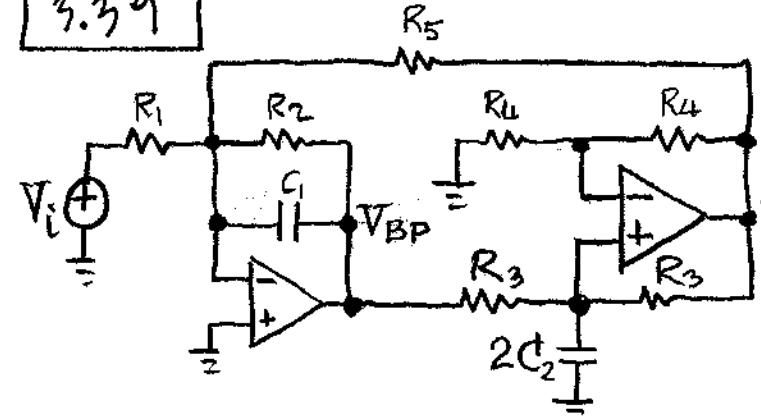
$$f_z = f_0 \left[\frac{\alpha}{\beta} \frac{H_{0LP}}{H_{0HP}}\right]^{1/2}.$$

(b) Let $C = 10\,nF$. Then, $R = 15.8\,k\Omega$. $R_2/R_1 = 2(Q-1) = 18$. Let $R_1 = 10\,k\Omega$, $R_2 = 180\,k\Omega$. Since $H_{0LP}/H_{0HP} = 1$ for the noninverting state-variable filter, it follows that $\alpha/\beta = (f_z/f_0)^2 = 2^2$. Moreover, since the dc gain of the notch response is $\alpha H_{0LP} = \alpha/Q$, it follows that to achieve unity dc gain we need $\alpha = 10$. Thus, we need a summing amplifier such that $V_N = -(10 V_{LP} + 2.5 V_{HP})$. This is shown below.

3.25



3.39



$V_{LP} = +\frac{1}{sR_3C_2}V_{BP}$

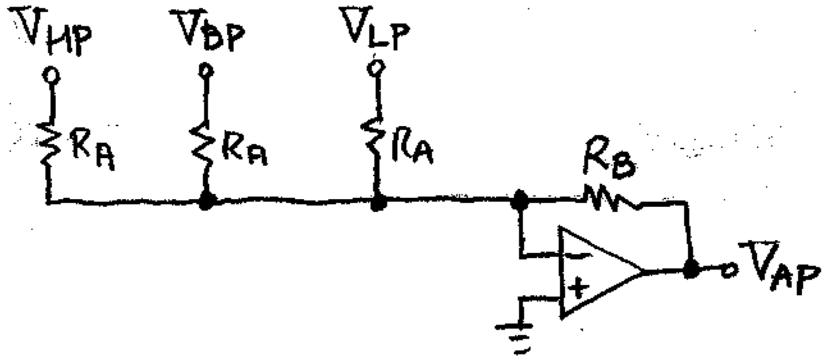
$$V_{BP} = -\frac{1}{sR_1C_1}V_i - \frac{1}{sR_2C_1}V_{BP} - \frac{1}{sR_5C_1}\left(+\frac{1}{sR_3C_2}V_{BP}\right)$$

$$\frac{V_{BP}}{V_i} = -\frac{(R_2/R_1) \times s R_3 R_5 C_1 C_2 / R_2 C_1}{1 + s^2 R_3 R_5 C_1 C_2 + s R_3 R_5 C_1 C_2 / R_2 C_1}$$

$\omega_0 = 1/\sqrt{R_3R_5C_1C_2}$, $Q = R_2\sqrt{C_1}/\sqrt{R_3R_5C_2}$, $H_{0BP} = -\frac{R_2}{R_1}$,
$H_{0LP} = -R_5/R_1$.

Let $C_1 = C_2 = 1\,nF$, $R_3 = R_5 = \frac{1}{2\pi \times 10^4 \times 10^{-9}} =$ $15.8\,k\Omega$, $R_2 = 80.6\,k\Omega$, $R_1 = 15.8\,k\Omega = R_4$.

$\boxed{3.40}$ Use the filter of Fig. 3.34 with $C_1 = C_2 = 10\,nF$, $R_6 = R_7 = 1/(2\pi 10^3 \times 10^{-8}) = 15.8\,k\Omega$, $R_1 = 10.0\,k\Omega$, $R_2 = 20.0\,k\Omega$, $R_3 = R_4 = R_5 = 10.0\,k\Omega$. Then the circuit gives $V_{HP} = -1 H_{HP} V_i$, $V_{BP} = 1 H_{BP} V_i$, and $V_{LP} = -1 H_{LP} V_i$, which we combine as:



$$\frac{V_{AP}}{V_i} = -\frac{R_B}{R_A}\left(-H_{HP} + H_{BP} - H_{LP}\right) = \frac{R_B}{R_A} H_{AP}.$$

Use $R_A = R_B = 10.0\,k\Omega$.

$\boxed{3.41}$ (3.92a):

$$S_{1/x}^{y} = \frac{\partial y}{\partial(1/x)}\frac{1/x}{y} = \frac{\partial y}{\partial x}\frac{\partial x}{\partial(1/x)}\frac{1/x}{y} = -\frac{\partial y}{\partial x}\frac{x}{y} = -S_x^y$$

Likewise, $S_x^{1/y} = -S_x^y$.

(3.92b):

$$S_x^{y_1 y_2} = \frac{\partial(y_1 y_2)}{\partial x}\frac{x}{y_1 y_2} = \left(y_2\frac{\partial y_1}{\partial x} + y_1\frac{\partial y_2}{\partial x}\right)\frac{x}{y_1 y_2}$$

$$= \frac{\partial y_1}{\partial x}\frac{x}{y_1} + \frac{\partial y_2}{\partial x}\frac{x}{y_2} = S_x^{y_1} + S_x^{y_2}$$

(3.92c):

$$S_x^{y_1/y_2} = S_x^{y_1 y_2^{-1}} = S_x^{y_1} + S_x^{y_2^{-1}} = S_x^{y_1} - S_x^{y_2}.$$

(3.92d):

(3.27)

$$S_x^{x^n} = \frac{x}{x^n}\frac{\partial x^n}{\partial x} = \frac{1}{x^{n-1}}\, n\, x^{n-1} = n.$$

(3.92e):

$$S_{x_1}^{y} = \frac{x_1}{y}\frac{\partial y}{\partial x_1} = \frac{x_2}{y}\frac{\partial y}{\partial x_2}\frac{x_1}{x_2}\frac{\partial x_2}{\partial x_1} = S_{x_2}^{y}\, S_{x_1}^{x_2}.$$

3.42 The denominator of $H(s)$ can be expressed as

$$D(s) = (s+a)(s+b) - Kcs$$

where $c>0$ is a suitable constant, and $-a$ and $-b$ are the zeros of D (poles of H) in the limit $K \to 0$. Rearranging as $D(s) = s^2 + s(a+b-Kc) + ab$ indicates $\omega_0 = 1/\sqrt{ab}$ and $Q = \sqrt{ab}/(a+b-Kc)$. Then,

$$S_K^Q = -\frac{-Kc}{a+b-Kc} = Q\left(\sqrt{\frac{a}{b}} + \sqrt{\frac{b}{a}}\right) - 1.$$

It is readily seen that the expression within parentheses is always ≥ 2, so $S_K^Q \geq 2Q-1$.

3.43 (a) $\omega_0 = 1/R\sqrt{C_1C_2}$, $Q = \sqrt{C_1/C_2}/3$.

(b) $S_{R_2}^{\omega_0} = S_{R_3}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = -1/2$; $S_{R_1}^{\omega_0} = 0$; $S_{R_2}^{Q} = S_{R_3}^{Q} = -1/6$; $S_{R_1}^{Q} = 1/3$; $S_{C_1}^{Q} = -S_{C_2}^{Q} = 1/2$.

3.44 From Eq. (3.79), $S_{R_4}^{\omega_0} = S_{R_6}^{\omega_0} = S_{R_7}^{\omega_0} = -S_{R_5}^{\omega_0} = S_{C_1}^{\omega_0} = S_{C_2}^{\omega_0} = 1/2$.

(3.28)

Writing $Q = A(1 + R_2/R_1)$ gives

$$S^Q_{R_2} = -S^Q_{R_1} = \frac{R_2}{Q}\frac{\partial Q}{\partial R_2} = \frac{R_2}{Q}\frac{A}{R_1} = \frac{1}{1 + R_1/R_2} = \frac{1}{1 + 1/299} \cong +1.$$

Writing $\frac{1}{Q} = B\left(1 + \frac{R_5}{R_3} + \frac{R_5}{R_4}\right)$ gives

$$S^Q_{R_3} = -S^{1/Q}_{R_3} = -QR_3\frac{\partial(1/Q)}{\partial R_3} = \frac{1}{1 + R_3/R_4 + R_3/R_5} = +\frac{1}{3}.$$

Writing $Q = D\sqrt{R_6 C_1 / R_7 C_2}$ gives

$$S^Q_{R_6} = S^Q_{R_7} = -S^Q_{R_7} = -S^Q_{C_2} = 1/2.$$

Writing $Q = E\frac{R_5^{1/2}}{1 + R_5/R_3 + R_5/R_4}$ gives

$$S^Q_{R_5} = \frac{1}{2}\,\frac{1 - R_5/R_3 - R_5/R_4}{1 + R_5/R_3 + R_5/R_4} = -\frac{1}{6}$$

Writing $\frac{1}{Q} = F\left(1 + \frac{R_5}{R_3} + \frac{R_5}{R_4}\right)\sqrt{R_4}$ gives

$$S^Q_{R_4} = -S^{1/Q}_{R_4} = -\frac{1}{2}\,\frac{1 + R_5/R_3 - R_5/R_4}{1 + R_5/R_3 + R_5/R_4} = -\frac{1}{6}$$