4.1

$\boxed{4.1}$ (a) $A(\omega) = 10\log_{10}[1+\varepsilon^2(\omega/\omega_c)^{2n}]$.
$A_{max} = 1\,dB \Rightarrow A(\omega_c) = 10\log_{10}(1+\varepsilon^2) = 1 \Rightarrow \varepsilon = 0.5088$
$A_{min} = 10\log_{10}[1+\varepsilon^2(\omega_s/\omega_c)^{2n}] \Rightarrow \varepsilon^2(\omega_s/\omega_c)^{2n} =$
$10^{A_{min}/10} - 1 \Rightarrow 1.2^{2n} = (10^{20/10}-1)/0.5088^2 \Rightarrow$
$n = 16.3 \Rightarrow$ use $n = 17$.

(b) $A(\omega_s) = 10\log_{10}[1+0.5088^2(1.2)^{34}] =$
21.09 dB.

(c) $\varepsilon^2 = (10^{20/10}-1)/(1.2)^{34} \Rightarrow \varepsilon = 0.4485$
$A_{max} = 10\log_{10}(1+0.4485^2) = 0.7959$ dB.

$\boxed{4.2}$ $A_{max} = 1\,dB \Rightarrow A(\omega_c) = 10\log_{10}(1+\varepsilon^2) \Rightarrow$
$\varepsilon = 0.5088$. $A(\omega_s) = 10\log_{10}[1+\varepsilon^2 C_n^2(\omega_s/\omega_c)]$
$= 10\log_{10}[1+0.5088^2\cosh^2(n\cosh^{-1}2)]$
$= 10\log_{10}[1+0.5088^2\cosh^2(1.317n)]$. Trying out different values of n, we find that $A(\omega_s) = 11.36$ dB for $n=2$, and $A(\omega_s) = 22.46$ dB for $n=3$. We thus choose $n=3$, which is substantially less than $n=7$ of Example 4.1.

$\boxed{4.3}$ $|H(j\omega)| = \dfrac{1}{\sqrt{1+\varepsilon^2\cos^2[7\cos^{-1}(\omega/\omega_c)]}}$

Peaks occur for $\cos[7\cos^{-1}(\omega/\omega_c)] = 0$, valleys for $\cos[7\cos^{-1}(\omega/\omega_c)] = \pm 1$.

4.2

Peaks: $7\cos^{-1}(\omega_p/\omega_c) = 2(k+1)\frac{\pi}{2}$, $k=0,1,2,3$.

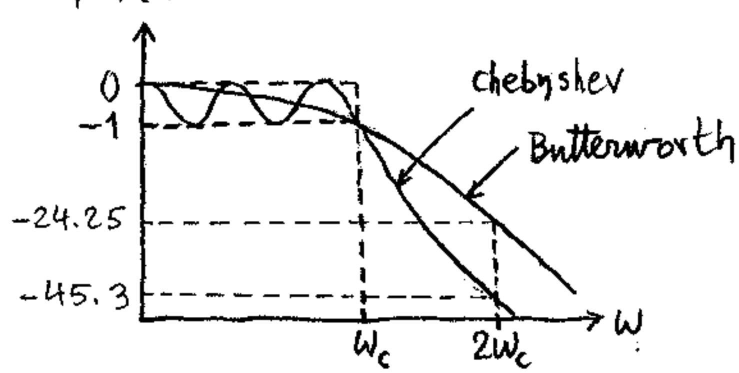
$\omega_p = \omega_c \cos\left[(2k+1)\frac{\pi}{14}\right] = 0.975\omega_c,\ 0.7818\omega_c,\ 0.434\omega_c,\ 0.$

Valleys: $7\cos^{-1}(\omega_v/\omega_c) = k\pi$, $k=0,1,2,3$.

$\omega_v = \omega_c \cos\frac{k\pi}{7} = \omega_c,\ 0.901\omega_c,\ 0.623\omega_c,\ 0.223\omega_c$.

$A_{max} = 0.5$ dB $\Rightarrow 0.5 = 10\log_{10}(1+\varepsilon^2) \Rightarrow \varepsilon = 0.3493$; $7\cosh^{-1}2 = 9.219$; $A(2\omega_c) = 10\times\log[1+\varepsilon^2\cosh^2 9.219] = 64.9$ dB. $7\cos^{-1}10 = 20.95$; $A(10\omega_c) = 10\log_{10}[1+\varepsilon^2\cosh^2 20.95] = 166.8$ dB.

4.4 (a)



(b) $A_{max} = 1$ dB $\Rightarrow 10\log_{10}(1+\varepsilon^2) = 1 \Rightarrow \varepsilon = 0.5088$.

Butterworth: $A(2\omega_c) = 10\log_{10}\left[1+\varepsilon^2\times 2^{2\times 5}\right] = 24.25$ dB.

Chebyshev: $A(2\omega_c) = 10\log_{10}\left[1+\varepsilon^2\cosh^2(5\cosh^{-1}2)\right] = 45.3$ dB.

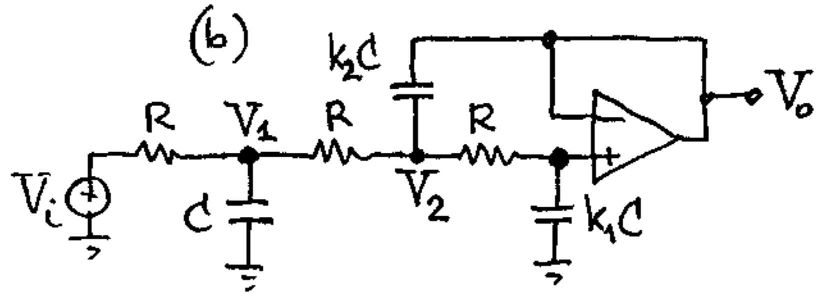
(4.3)

$\boxed{4.5}$ (a)

$$|H|^2 = \frac{1}{[1-2(\omega/\omega_c)^2]^2 + [2(\omega/\omega_c) - (\omega/\omega_c)^3]^2}$$

$$|H|^2 = \frac{1}{1+(\omega/\omega_c)^6}$$

(b)



$$V_o = \frac{1}{1+j\omega k_1 RC} V_2 \Rightarrow V_2 = (1+j\omega k_1 RC) V_o.$$

$$\frac{V_i - V_1}{R} = j\omega C V_1 + \frac{V_1 - V_2}{R}, \text{ that is,}$$

$$V_i = (2+j\omega RC) V_1 - V_2.$$

$$\frac{V_1 - V_2}{R} = j\omega k_2 C (V_2 - V_o) + \frac{V_2 - V_o}{R}, \text{ that is,}$$

$$V_1 = V_2 (j\omega k_2 RC + 2) - V_o (j\omega k_2 RC + 1).$$

Eliminating V_1 and V_2 and collecting,

$$H = \frac{1}{1 - j(\omega RC)^3 k_1 k_2 - (\omega RC)^2 2k_1(k_2+1) + j\omega RC(3k_1+1)}.$$

Letting $(\omega RC)^3 k_1 k_2 = (\omega/\omega_c)^3$ yields $\omega_c = \frac{1}{(k_1 k_2)^{1/3} RC}$.

Letting $(\omega RC)^2 2k_1(k_2+1) = 2(\omega/\omega_c)^2$ and $\omega RC(3k_1+1) = 2$ yields, respectively,

$\frac{k_1(k_2+1)}{(k_1k_2)^{2/3}} = 1$ and $\frac{3k_1+1}{(k_1k_2)^{1/3}} = 2$. Eliminat-

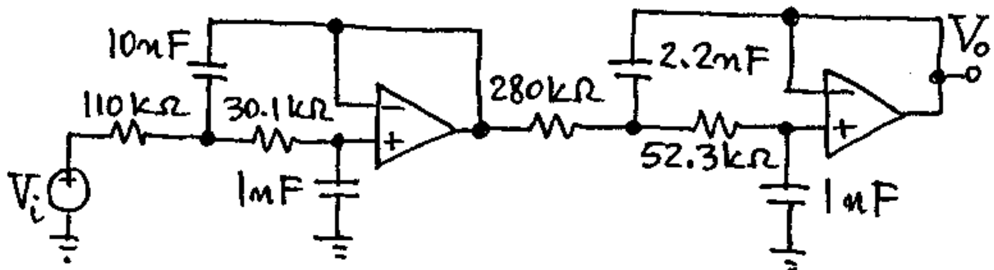
(4.4)

ing the product $k_1 k_2$ yields $(3k_1+1)^2 = 4k_1(k_2+1)$. Eliminating the product $k_1 k_2$ once more, $27k_1^3 + 9k_1^2 + 5k_1 - 1 = 0$. Solving by iteration starting with $k_1 = 0$ yields $k_1 = 0.14537$. Backsubstituting, $k_2 = 2.5468$.

(c) Let $R = 100\ \mathrm{k}\Omega$. Then, $C = 1/[2\pi \times 10^3 \times 10^5 \times (0.14537 \times 2.5468)^{1/3}] = 2.216\ \mathrm{nF}$, $mC = 322.2\ \mathrm{pF}$, $nC = 5.645\ \mathrm{nF}$.

$\boxed{4.6}$ (a)
$$|H|^2 = \frac{1}{[1-(\omega/\omega_c)^2]^2 + (2-\sqrt{2})(\omega/\omega_c)^2} \times \frac{1}{[1-(\omega/\omega_c)^2]^2 + (2+\sqrt{2})(\omega/\omega_c)^2} = \frac{1}{1+(\omega/\omega_c)^8}.$$

(b)



(4.5)

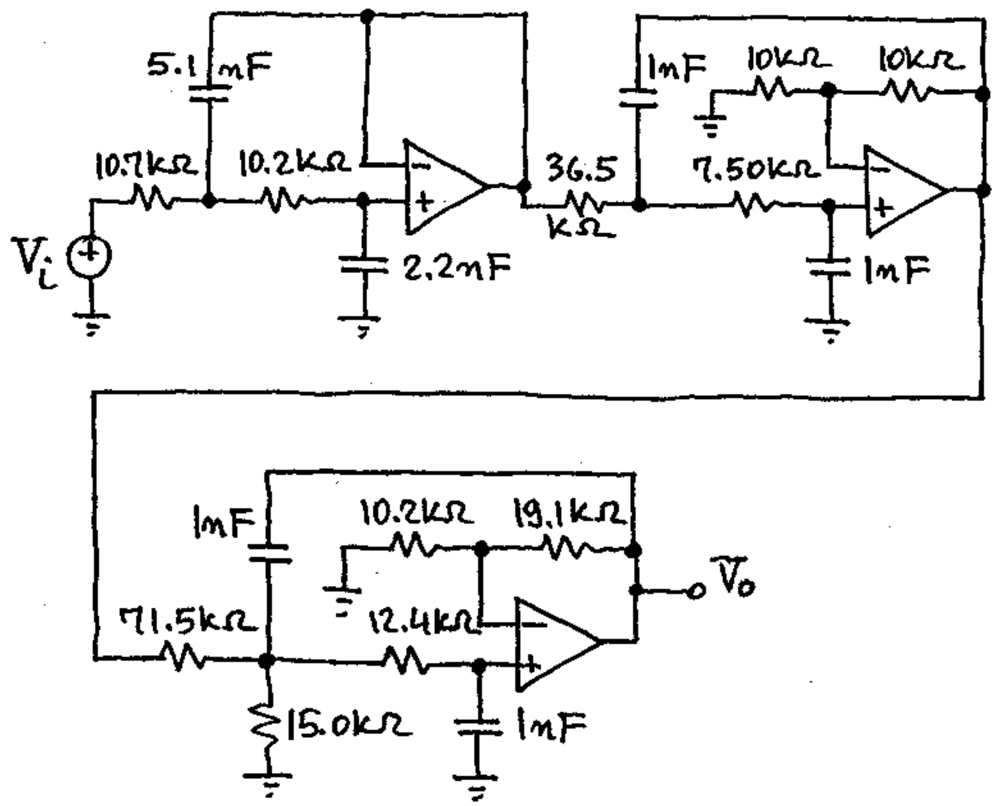
4.7 1st Stage: leave it unchanged since the component spread is low.

2d Stage: Use $K=2$. Then, $Q=\sqrt{m}=2.20 \Rightarrow m=4.84$. Let $C=1\,nF$. Then, $R=1/(2\pi Q f_0 C)=1/(2\pi\times 2.2\times 9.56\times 10^3\times 10^{-9})=7.567\,k\Omega$, $mR=36.62\,k\Omega$.

3d Stage: Use $m=n=1$, $C=1\,nF$. Then, $R=1/(2\pi f_0 C)=1/(2\pi\times 12.74\times 10^3\times 10^{-9})=12.492\,k\Omega$. $K=3-1/Q=3-1/8=2.875 \Rightarrow R_B/R_A=1.875$. Let $R_A=10.2\,k\Omega$, $R_B=19.1\,k\Omega$.

Dc Gain: $H_0=K_2\times K_3=2\times 2.875=5.75$. To achieve $H_0=1$, split the first resistor of the third stage into two resistors, R_1 and R_2, such that $R_2/(R_1+R_2)=1/5.75$ and $R_1/\!/R_2=12.49\,k\Omega$. Thus, $R_1R_2/(R_1+R_2)=R_1/5.75=12.49 \Rightarrow R_1=71.83\,k\Omega$ and $R_2=15.12\,k\Omega$. To calibrate Q_3, it may be advisable to make the $19.1\text{-}k\Omega$ resistor adjustable.

(4.6)



4.8 Retracing the steps of Example 3.10, we find the following values:

C=2nF, nC=3.3nF, R=12.51kΩ, mR=19.43kΩ.

C=1.2nF, nC=10nF, R=3.96kΩ, mR=9.41kΩ.

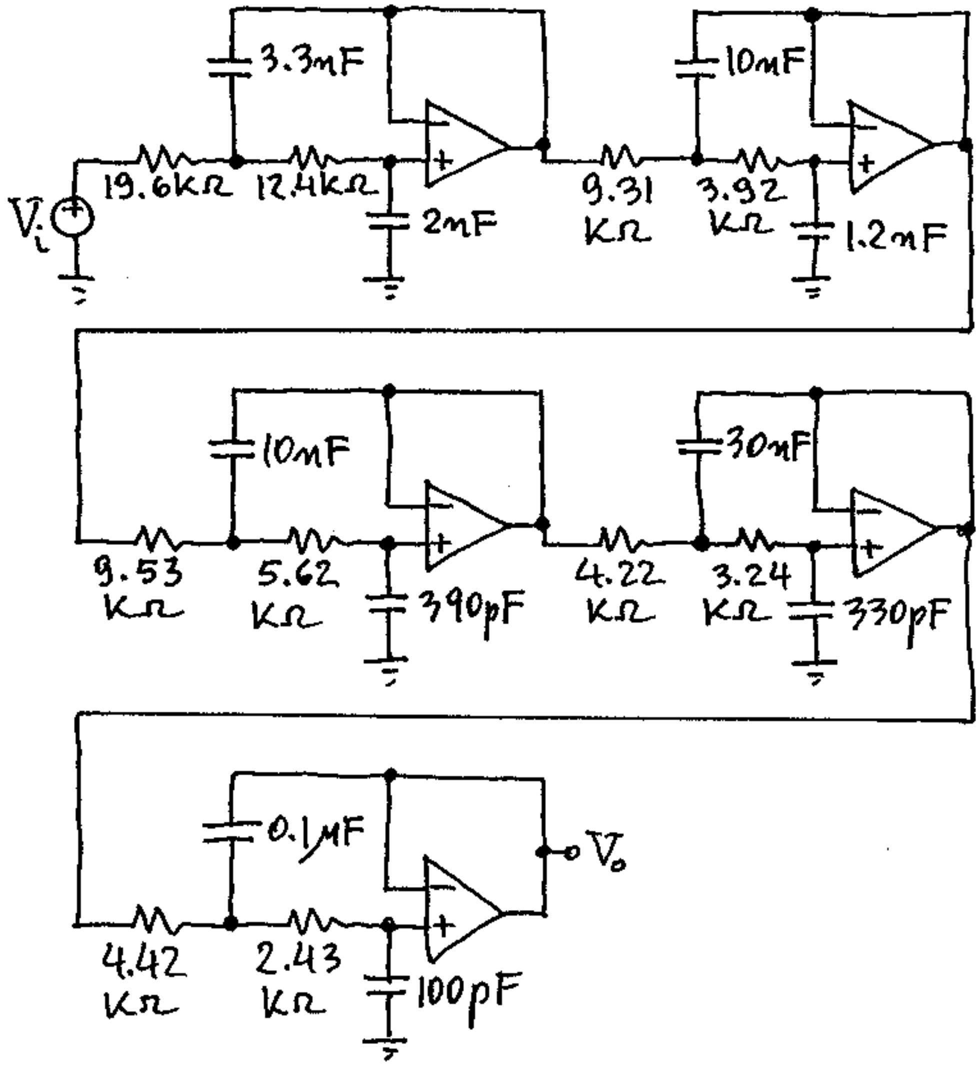
C=390pF, nC=10nF, R=5.57kΩ, mR=9.465kΩ.

C=330pF, nC=30nF, R=3.22kΩ, mR=4.21kΩ.

C=100pF, nC=0.1μF, R=2.456kΩ, mR=4.478kΩ.

The implementation with 1% rounded off values is shown in the figure.

(4.7)

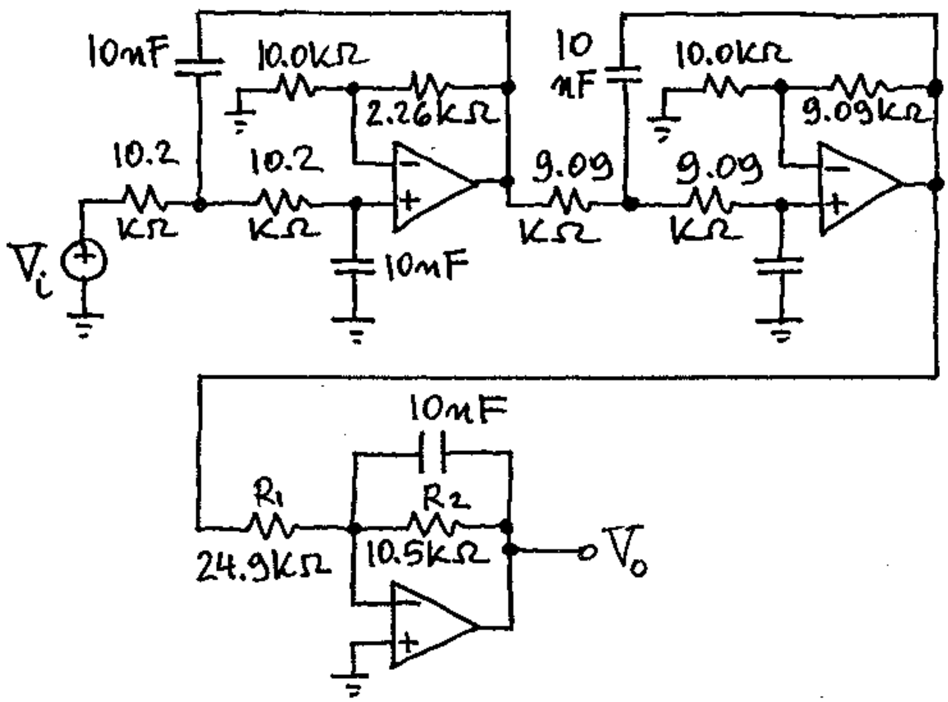


4.9 Using Table 4.1, we find:

1st Stage: $f_0 = 1.561$ KHz, $Q = 0.564$.
Let $C = 10$ nF. Then, $R = 1/(2\pi \times 1.561 \times 10^{-5}) = 10.2$ kΩ. $K = 3 - 1/Q = 3 - 1/0.564 = 1.227$
$\Rightarrow R_B/R_A = 0.227$. Use $R_A = 10.0$ kΩ, $R_B = 2.26$ kΩ.

2d stage: $f_0 = 1.760$ KHz, $Q = 0.917$.
$C = 10$ nF, $R = 9.04$ KΩ, $R_A = 10.0$ KΩ, $R_B = 9.09$ KΩ.

(4.8) 3d Stage: $f_0 = 1.507$ kHz, $C = 10$ nF, $R_2 = 1/(2\pi f_0 C) = 1/(2\pi \times 1.507 \times 10^{-5}) = 10.56$ kΩ. For 0 dB dc gain we need $R_2/R_1 \times K_1 \times K_2 = 1 \Rightarrow R_1 = 10.56 \times 1.227 \times 1.909 = 24.73$ kΩ.



$\boxed{4.10}$ From Table 4.1, we need three second-order stages with $Q_1 = 0.555$, $Q_2 = 0.802$, $Q_3 = 2.247$, and a first-order stage. Use $C = 1$ nF throughout. Then:

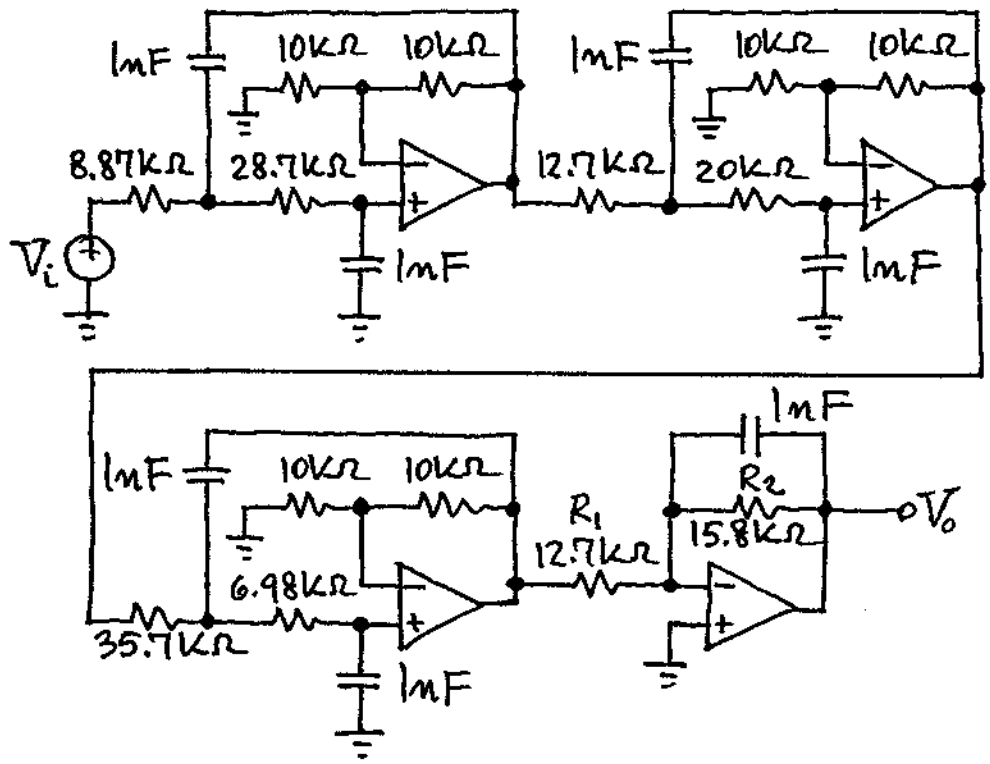
<u>1st stage</u>: $Q = \sqrt{m} = 0.555 \Rightarrow m = 0.308$; $R = 1/2\pi Q f_0 C = 1/(2\pi \times 0.555 \times 10^4 \times 10^{-9}) = 28.67$ kΩ; $mR = 8.83$ kΩ. Similarly:

<u>2nd stage</u>: $R = 19.84$ kΩ, $mR = 12.76$ kΩ.

<u>3rd stage</u>: $R = 7.083$ kΩ, $mR = 35.76$ kΩ

(4.9)

1st-order stage: $C = 1\,nF$, $R_2 = 1/(2\pi f_0 C) = 1/(2\pi \times 10^4 \times 10^{-9}) = 15.9\,k\Omega$. $(R_2/R_1) \times 2 \times 2 \times 2 = 20\,dB = 10 \Rightarrow R_1 = (8/10) R_2 = 12.73\,k\Omega$.



4.11	From Table 4.1 we see that we need

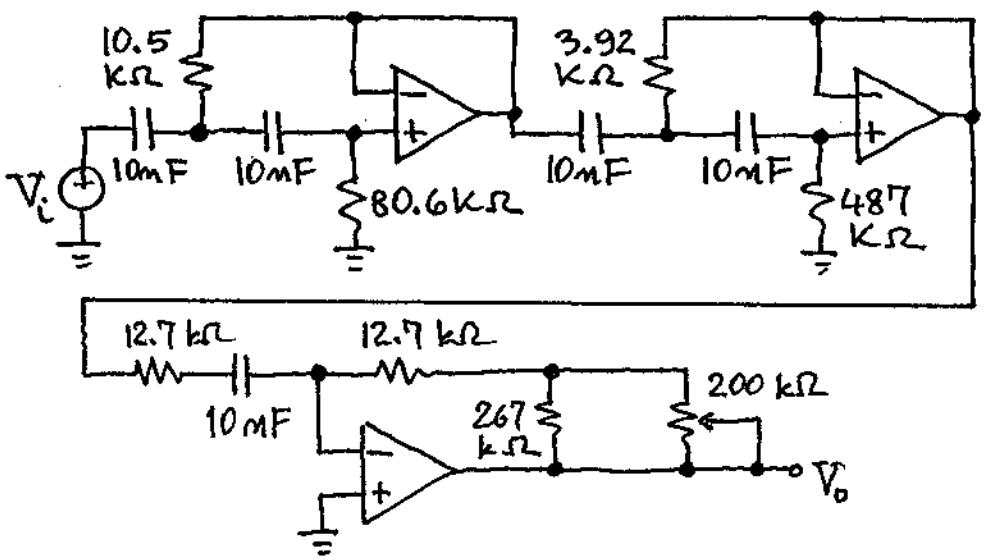
two second-order stages with $f_{01} = 360/0.994$ $362.17\,Hz$, $Q_1 = 5.556$, and $f_{02} = 360/0.655 = 549.62\,Hz$, $Q_2 = 1.399$; and a first-order stage with $f_{03} = 360/0.289 = 1{,}245.7\,Hz$.

1st stage: Let $C = 10\,nF$ and $n = 1$. Then, $Q = (\sqrt{m})/2 = 5.556 \Rightarrow m = 123.48$ $R = 1/(2\pi \sqrt{m} f_{01} C) = 1/(2\pi \times 2 \times 5.556 \times 362.17 \times 10^{-8}) = 3.955\,k\Omega$, $mR = 488.3\,k\Omega$.

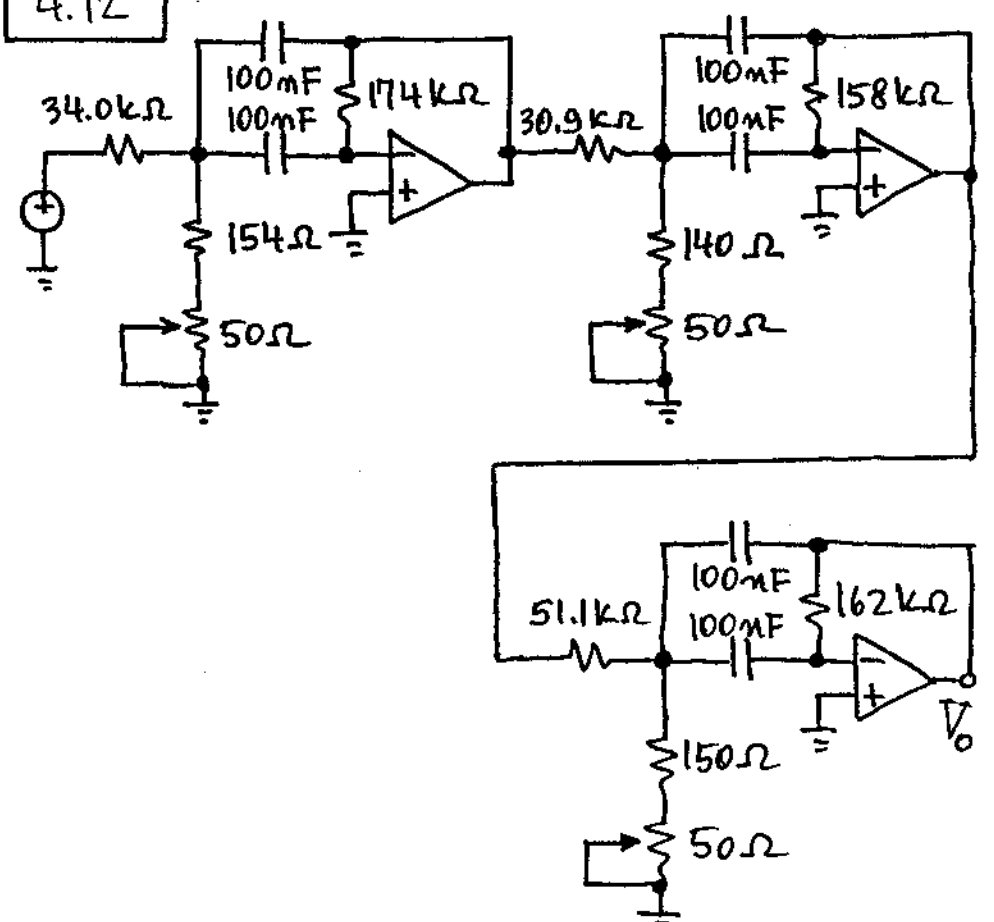
2d Stage: $C = 10\,nF$, $R = 10.35\,k\Omega$, $mR = 81.02\,k\Omega$.

(4.10)

3d stage: Let $C_1 = 10\,nF$. Then, $R_1 = 1/(2\pi f_{03} C) = 1/(2\pi \times 1{,}245.7 \times 10^{-8}) = 12.77$ kΩ. 20 dB high-frequency gain ⇒ $R_1 \le R_2 \le 10R_1$, $R_1 = 12.7$ kΩ, $10R_1 = 127$ kΩ.



4.12



(4.11)

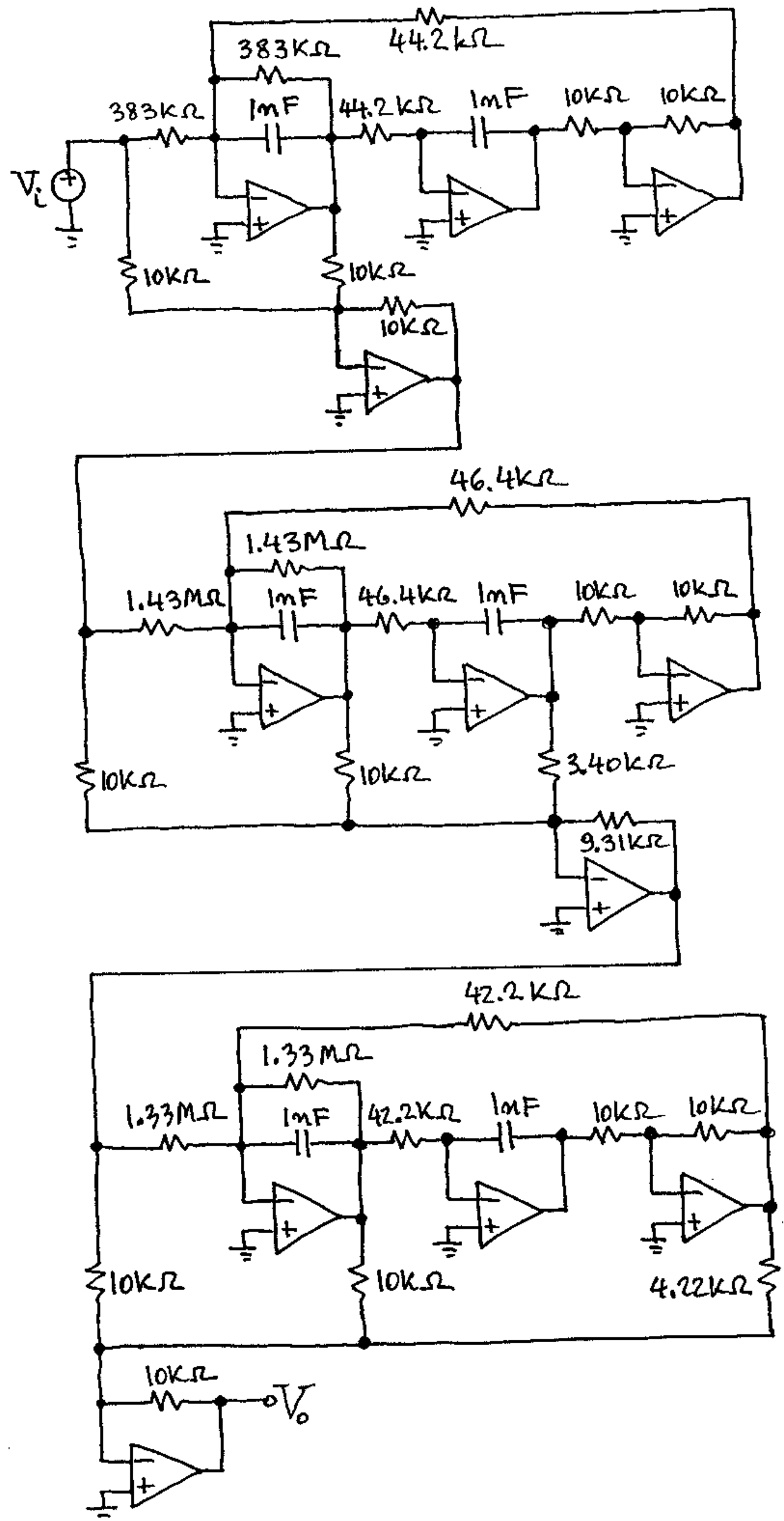
$\boxed{4.13}$ Use $C = 1\,nF$ throughout.

1st stage: $R = 1/(2\pi f_{01} C) = 1/(2\pi \times 3460.05 \times 10^{-9}) = 45.99\ K\Omega$. $R_1 = Q_1 R = 31.4 \times 45.99 = 1.444\ M\Omega$. Let $R_2 = 10\ k\Omega$. Then, $R_4 = (R_2/Q_1) f_{01}^2 / |f_{01}^2 - f_{z1}^2| = (10^4/31.4) \times 3460.05^2 / |3460.05^2 - 3600^2| = 3.859\ K\Omega$. $R_5 = R_2 (f_{01}/f_{z1})^2 = 10^4 (3460.05/3600)^2 = 9.238\ K\Omega$.

2d stage: $R = 42.491\,K\Omega$, $R_1 = 1.334\ M\Omega$, $R_4 = 4.177\,K\Omega$, $R_5 = 10\,K\Omega$.

3d stage: $R = 1/(2\pi f_{03} C) = 1/(2\pi \times 3600 \times 10^{-9}) = 44.21\,K\Omega$. $R_1 = Q_3 R = 8.72 \times 44.21 = 385.5\ k\Omega$.

4.12


44.2kΩ
383KΩ
383KΩ
1mF
44.2KΩ
1mF
10KΩ
10KΩ
V_i
10KΩ
10KΩ
10KΩ
46.4KΩ
1.43MΩ
1.43MΩ
1mF
46.4KΩ
1mF
10KΩ
10KΩ
10KΩ
10KΩ
3.40KΩ
9.31KΩ
42.2KΩ
1.33MΩ
1.33MΩ
1mF
42.2KΩ
1mF
10KΩ
10KΩ
10KΩ
10KΩ
4.22KΩ
10KΩ
V_o

(4.13)

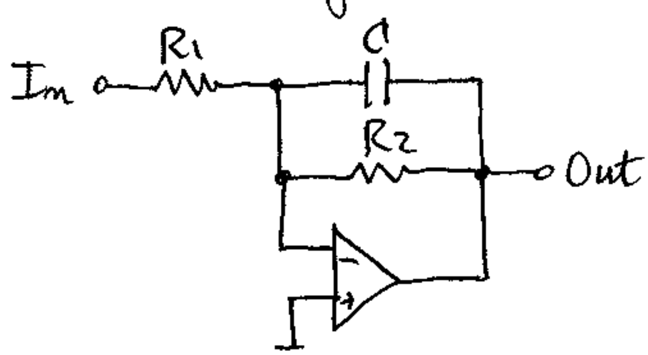
$\boxed{4.14}$ Using the FILDES Program, we find that a 7th order filter with the following individual-stage parameters

Stage No	f_0 (kHz)	Q
1	10.08	8.84
2	8.23	2.57
3	5.04	1.09
4	2.56	

will provide an attenuation of 64.9 dB at 20 kHz. For the first three stages use the unity-gain KRC configuration, for which

$$\omega_0 = \frac{1}{\sqrt{mn}\,RC} \qquad Q = \frac{\sqrt{mn}}{m+1} \qquad H_{0LP} = 1 \text{ V/V}.$$

For the last stage use the 1st order circuit



with $\omega_0 = \dfrac{1}{R_2 C}$ $\quad H_0 = -\dfrac{R_2}{R_1}$.

Pick $C = 0.1\,\mu F$, $R_2 = 62.17\ k\Omega$, $R_1 = 15.6\,k\Omega$.
For the other stages use the following:

(4.14) Stage No 1: $C = 1\,nF$, $mC = 330\,nF$,

$R = 688\,\Omega$, $mR = 1.098\,k\Omega$.

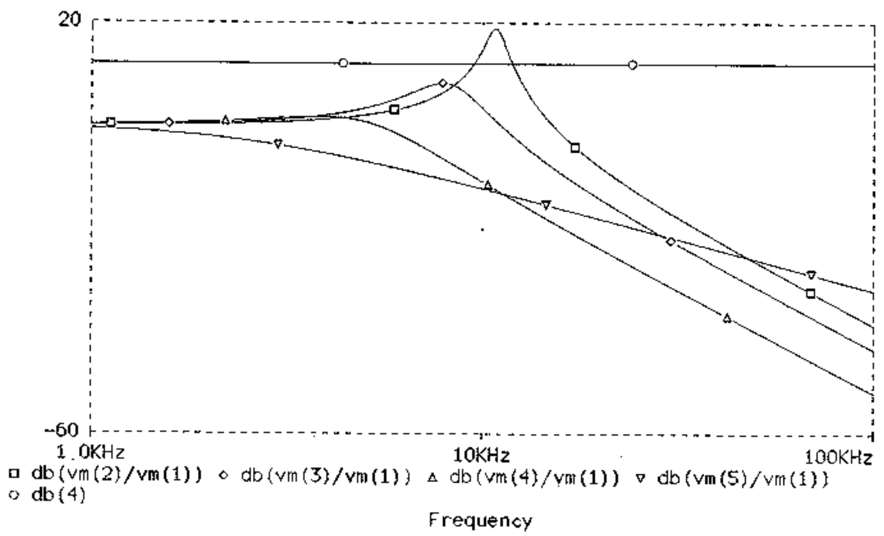
Stage No 2: $C = 1\,nF$, $nC = 33\,nF$,

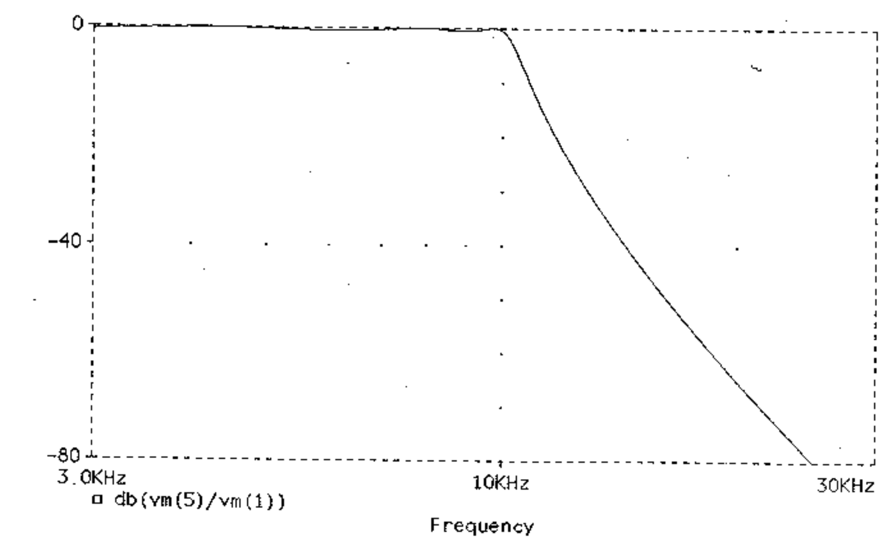
$R = 2.08\,k\Omega$, $mR = 5.44\,k\Omega$.

stage No 3: $C = 1\,nF$, $nC = 5\,nF$

$R = 11.26\,k\Omega$, $mR = 17.71\,k\Omega$

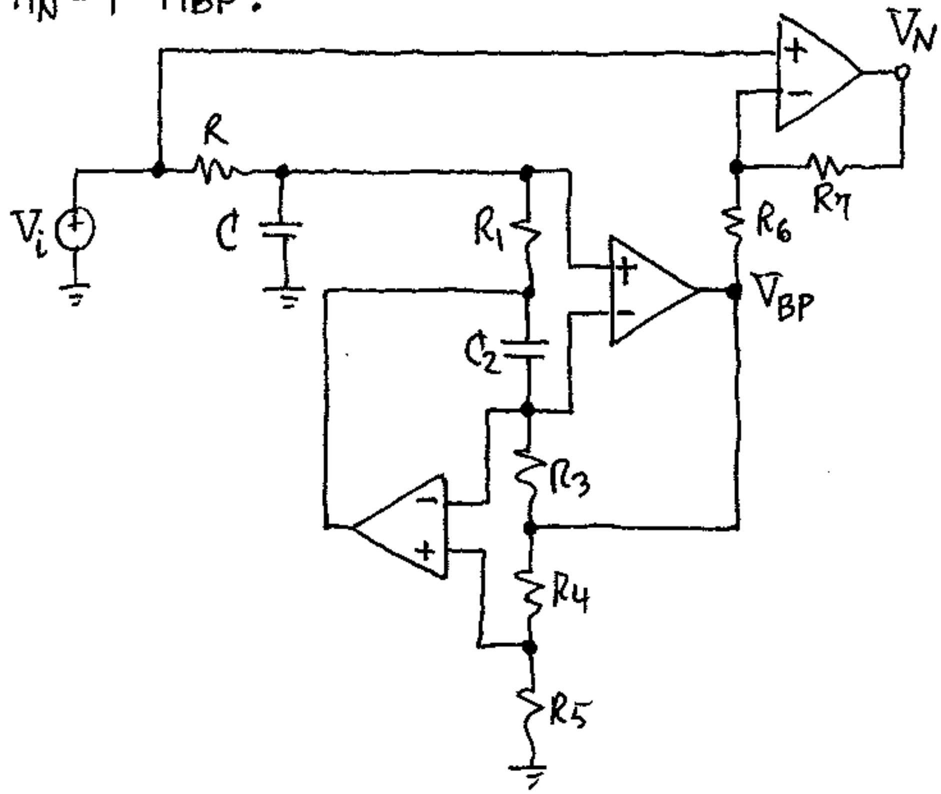
Computer simulation gives the following:





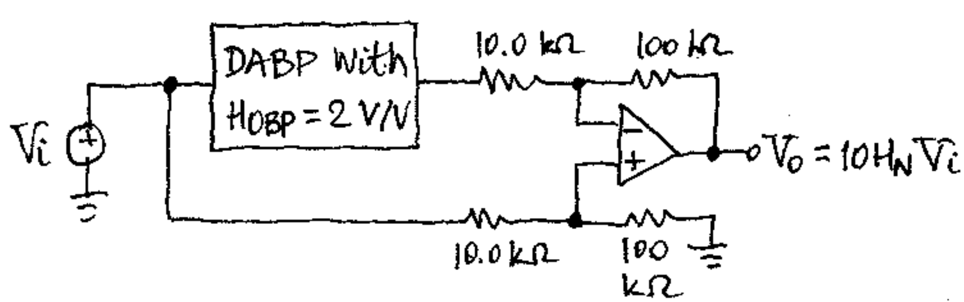
4.15

$\boxed{4.15}$ (a) Implement the notch response as $H_N = 1 - H_{BP}$.



Let $R_4 = R_5$, so that $V_{BP} = 2H_{BP}V_i$; let $R_6 = R_7$, so that $V_N = 2V_i - 2H_{BP}V_i = H_{0N}H_NV_i$, $H_{0N} = 2$ V/V. Let $C_2 = C = 0.1\,\mu F$. Then, $R_1 = R_3 = 13.26\ k\Omega$ (use $13.3\ k\Omega$), $R = 20R_1 = 267\ k\Omega$. Use $R_3 = R_4 = R_6 = R_7 = 10.0\ k\Omega$.

(b) Implement the AP response as $H_{AP} = 1 - 2H_{BP}$.

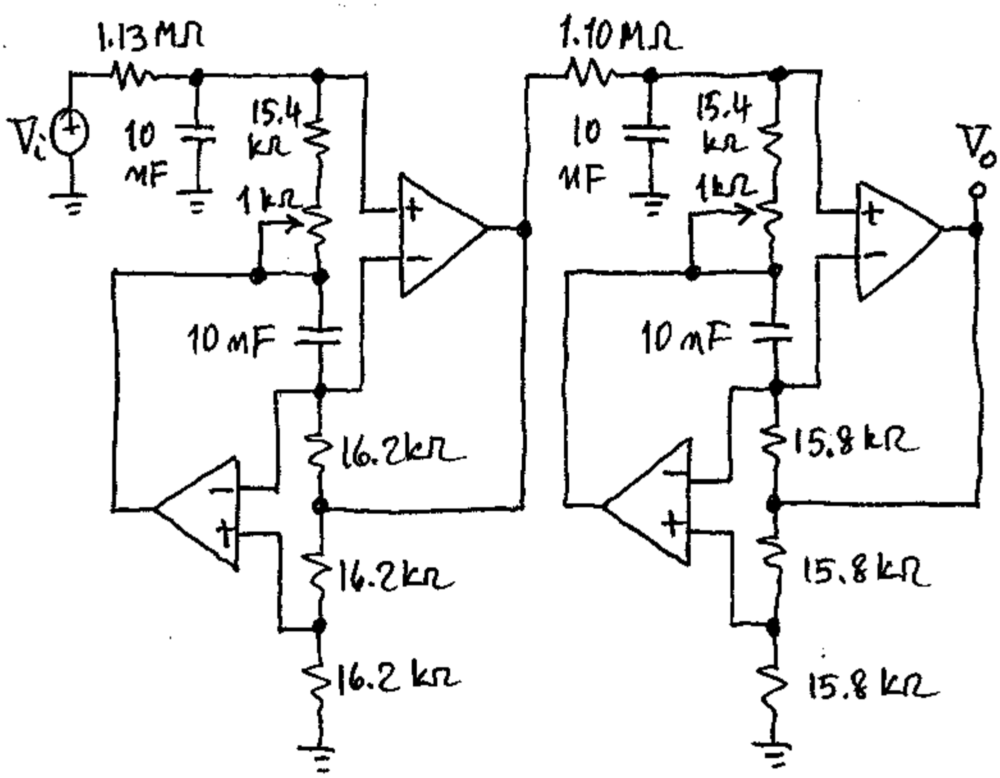


(4.16)

$\boxed{4.16}$ Use $C = 10$ nF throughout, as well as $R_1 = R_3 = R_4 = R_5$.

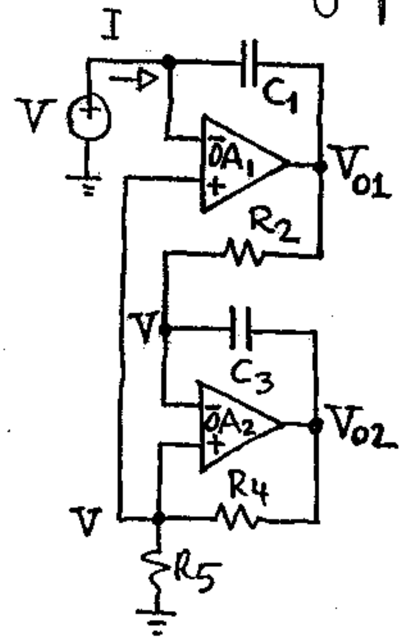
1st stage: $R_1 = 1/(2\pi f_{01} C) = 16.03$ kΩ; $R = QR_1 = 1.123$ MΩ.

2d stage: $R_1 = 15.8$ kΩ, $R = 1.107$ MΩ.



4.17

$\boxed{4.17}$ (a) By op amp action, $V_{p2} = V_{n2} = V_{p1} = V_{n1} = V$. By Ohm's law, $I = j\omega C_1 (V - V_{o1})$.



By KCL,

$$\frac{V_{o1} - V}{R_2} = j\omega C_3 (V - V_{o2}).$$

By the voltage divider,

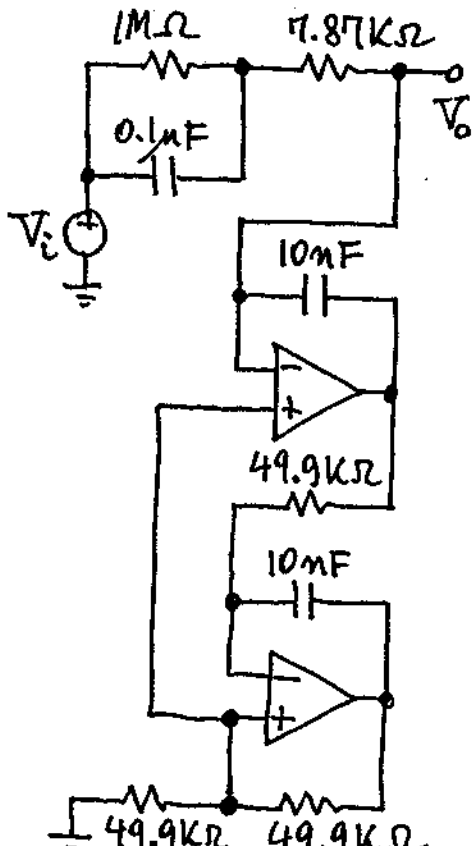
$$V_{p2} = V = \frac{R_5}{R_4 + R_5} V_{o2}.$$

Eliminating V_{o1} and V_{o2},

$$I = -\omega^2 \frac{C_1 R_2 C_3 R_4}{R_5} V = -\omega^2 D V.$$ Thus,

$$Z = V/I = 1/(-\omega^2 D), \quad D = C_1 R_2 C_3 R_4 / R_5.$$

(b) Let $C = 0.1\,\mu F$. Then,

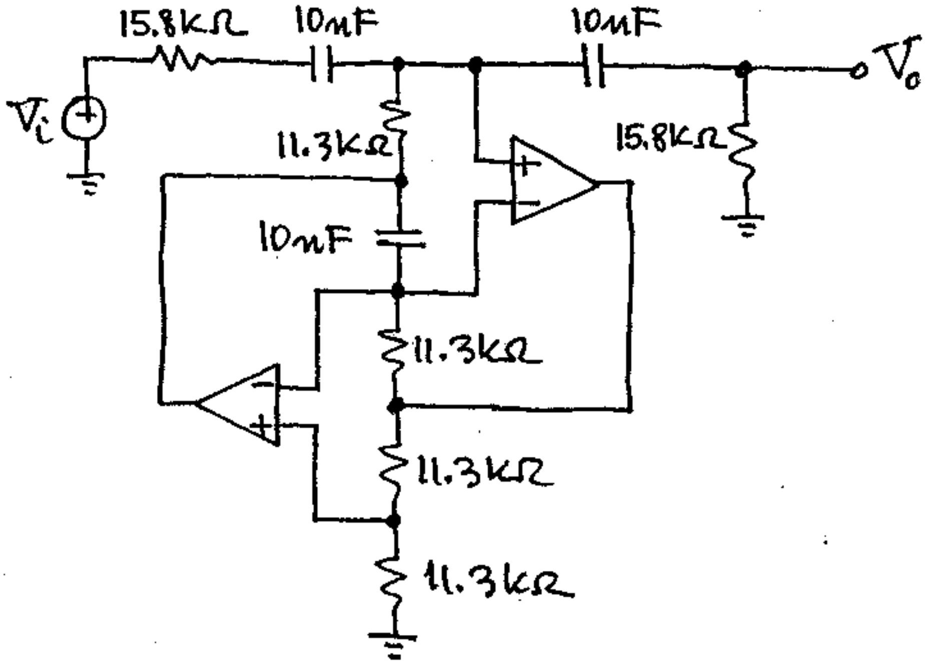


$D = C/(2\pi Q f_0) = 10^{-7}/(2\pi \times 4 \times 800) = 4.973 \times 10^{-12}\ s^2/\Omega$.

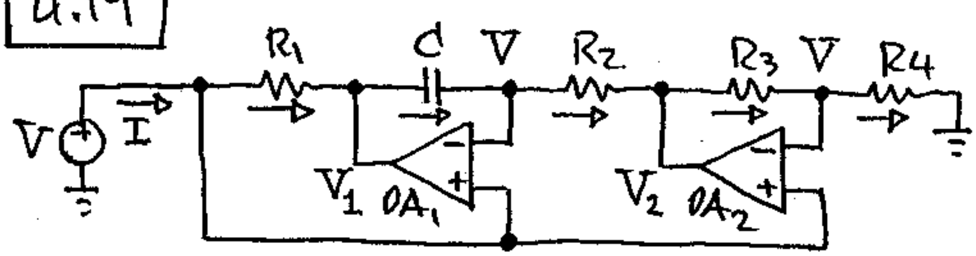
$R = D(Q/C)^2 = 4.973 \times 10^{-12} (4/10^{-7})^2 = 7.96\ K\Omega$ (use $7.87\ K\Omega$). Let $C_1 = C_3 = 10\,nF$, $R_2 = R_4 = R_5$. Then, $R_2 = D/C_1^2 = 4.973 \times 10^{-12}/(10^{-8})^2 = 49.7\,K\Omega$ (use $49.9\,K\Omega$.

(4.18)

$\boxed{4.18}$ Let $C = 10\,nF$. Then, $L = 1/[(2\pi f_0)^2 \times 2C] = 1/[(2\pi\times 10^3)^2 \times 2\times 10^{-8}] = 1.267\,H$. $R = (2\times 1.267/10^{-8})^{1/2} = 15.92\,k\Omega$. To find the component values for the GIC, impose equal resistors and equal, $10\,nF$, capacitances. Then, $R_k^2 \times 10\,nF = 1.267 \Rightarrow R_k = 11.25\,k\Omega$, $k = 1, 3, 4, 5$. Actual realization:



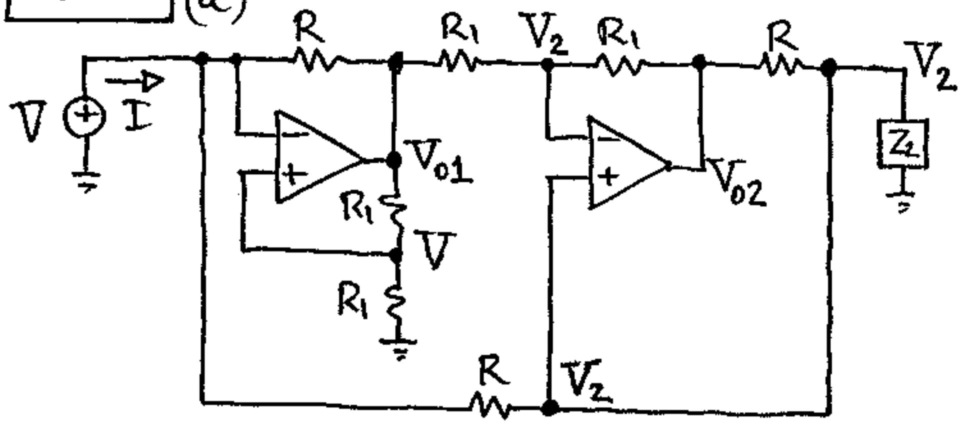
$\boxed{4.19}$



$I = \dfrac{V - V_1}{R_1}$; $j\omega C(V_1 - V) = \dfrac{V - V_2}{R_2}$; $V_2 = \left(1 + \dfrac{R_3}{R_4}\right) \times V$. Eliminating V_1 and V_2 and collecting, $V/I = j\omega L$, $L = (R_1 R_2 R_4/R_3)C$.

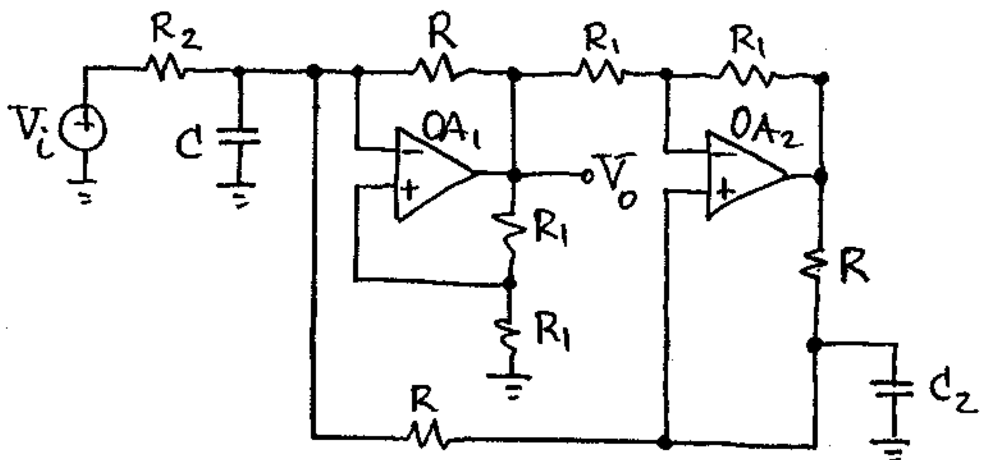
4.19

4.20 (a)



$V_{o1} = 2V$; $V_{o2} = -V_{o1} + 2V_2$; $I = \frac{V - V_{o1}}{R} + \frac{V - V_2}{R}$; $\frac{V - V_2}{R} = \frac{V_2 - V_{o2}}{R} + \frac{V_2}{Z_2}$. Eliminating V_{o1}, V_{o2}, and V_2 and collecting, $Z_1 = \frac{V}{I} = \frac{R^2}{Z_2}$.

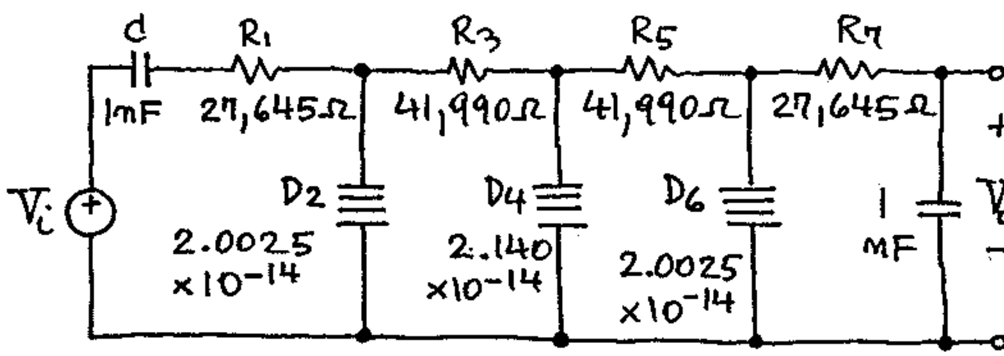
(b)



Let $C = 1\mu F$. Then, $L = 1/[(2\pi f_0)^2 C] = 1/[(2\pi)^2 10^{-6}] = 25.33$ KH. Use $C_2 = 1\mu F$, $R = \sqrt{L/C_2} = \sqrt{25{,}330 \times 10^6} = 158\ k\Omega$, $R_1 = 100\ k\Omega$, $R_2 = Q\sqrt{L/C} = 10\sqrt{25330 \times 10^6} = 1.58\ M\Omega$. Obtain V_o from OA_1's output, where the output impedance is low. Then, $H_{0BP} = 2$ V/V.

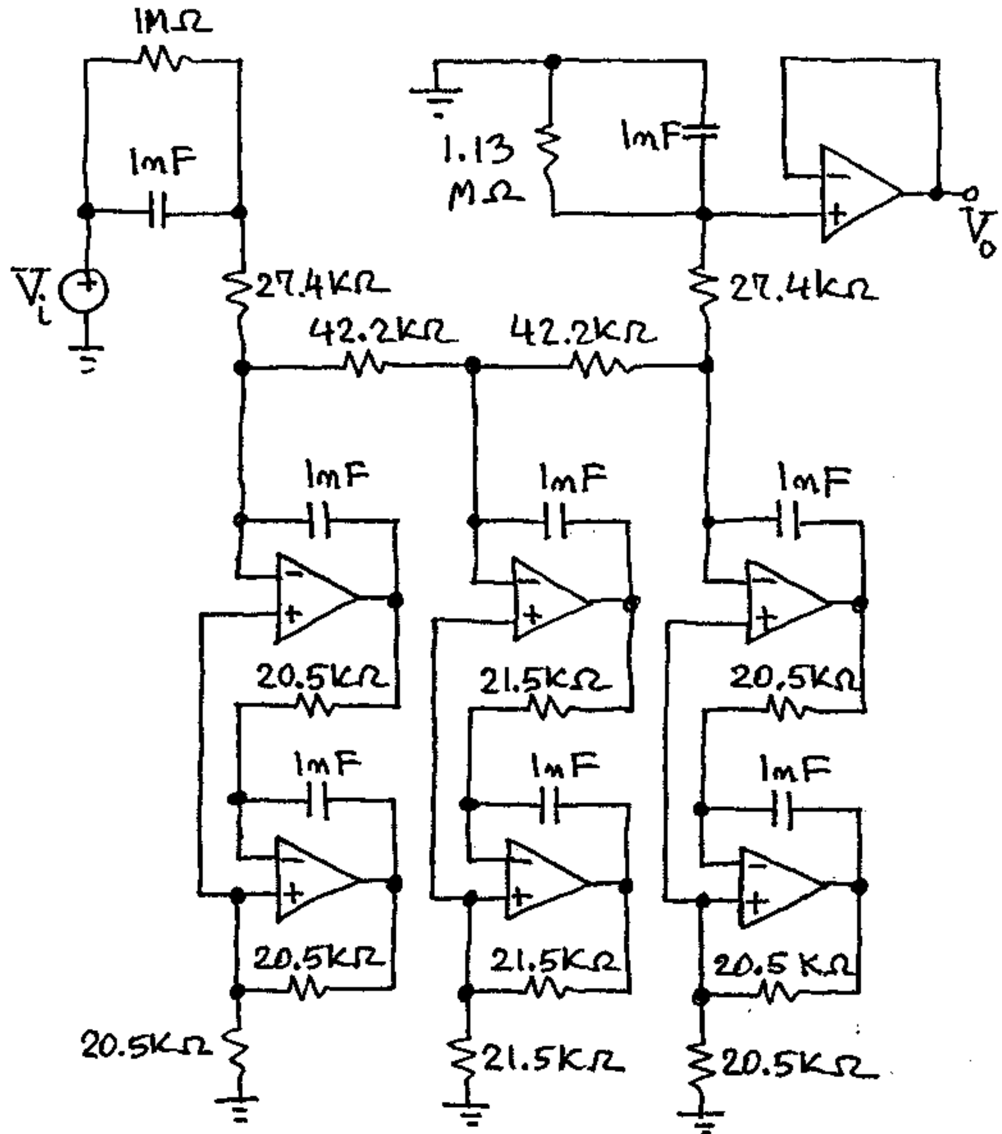
4.20

$\boxed{4.21}$ Use 1nF capacitors, so that $K_Z = 10^9$ Ω^2/s. Then, $C_{(new)} = 10^{-9} R_{(old)}$; $R_{(new)} =$ $15{,}915\, L_{(old)}$, $D_{(new)} = 1.592 \times 10^{-14}\, C_{(old)}$.
The transformed ladder is as follows:



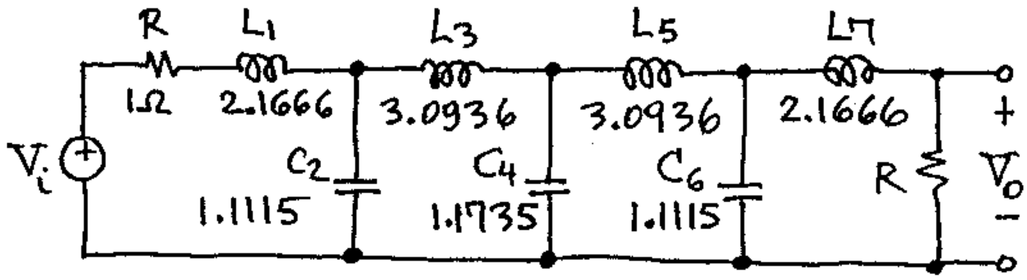
In the FDNRs let $C_1 = C_3 = 1nF$, $R_2 = R_5 = R_4$.
Then, $D = R_2 C_1^2 \Rightarrow R_2 = D/C_1^2 = 10^{18} D$. Thus, for D_2 and D_6 we have $R_2 = R_5 = R_4 = 10^{18} \times 2.0025 \times 10^{-14} = 20.025\ K\Omega$. For D_4 we have $R_2 = R_5 = R_4 = 10^{18} \times 2.140 \times 10^{-14} = 21.4\ K\Omega$.

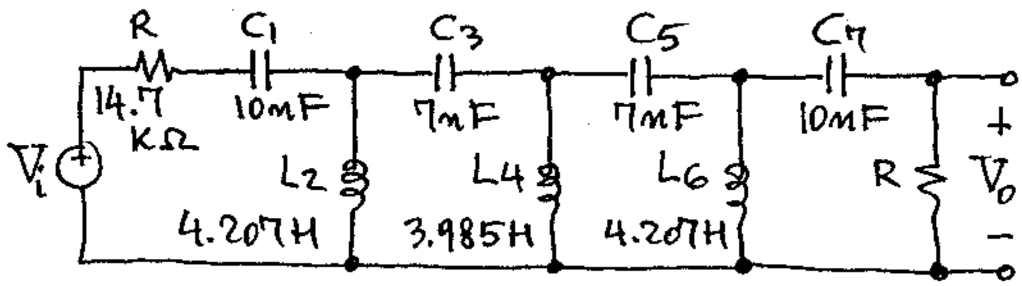
4.21


1MΩ
1mF
V_i
27.4KΩ
42.2KΩ
42.2KΩ
1.13 MΩ
1mF
27.4kΩ
V_o
1mF
1mF
1mF
20.5KΩ
21.5KΩ
20.5KΩ
1mF
1mF
1mF
20.5KΩ
21.5KΩ
20.5 KΩ
20.5KΩ
21.5KΩ
20.5KΩ

(4.22)

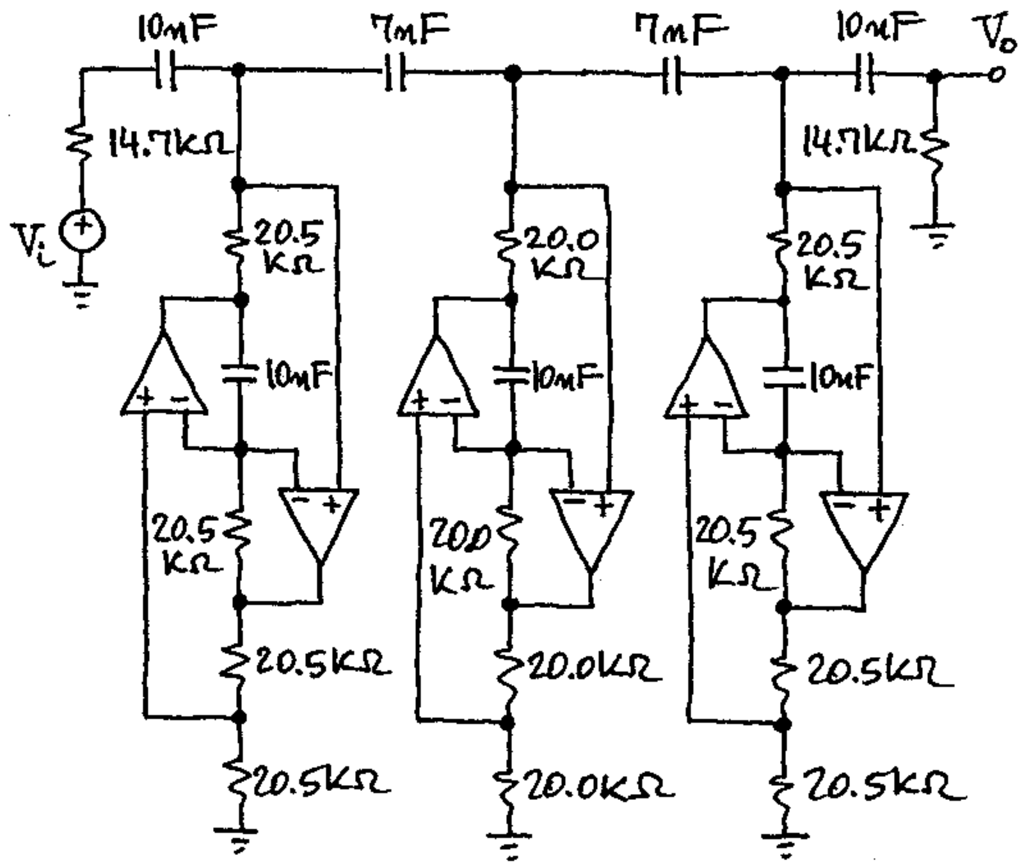
$\boxed{4.22}$ The prototype ladder is shown on top. Arbitrarily choose K_Z so that L_1 and L_7 are changed into 10 nF capacitors. Thus, $10\times10^{-9} = 1/(2\pi\times500\times K_Z\times 2.1666) \Rightarrow K_Z = 14{,}692$. $R_{(new)} = K_Z/R_{(old)} = 14.69$ KΩ. $C_1 = C_7 = 10$ nF. $C_3 = C_5 = 1/(2\pi\times500\times 14{,}692\times 3.0936) = 7$ nF. $L_2 = L_6 = 14{,}692/(2\pi\times500\times 1.1115) = 4.207$ H. $L_4 = 14{,}692/(2\pi\times 500\times 1.1735) = 3.985$ H. The high-pass ladder is shown at the bottom.





To find the element values in the GICs, impose equal resistances and $C = 10$ nF throughout. For the 1st and 3d GIC we obtain $R_k^2 = 4.207/10^{-8} \Rightarrow R_k = 20.51$ KΩ. For the 2d GIC, $R_k^2 = 3.985/10^{-8} \Rightarrow R_k = 19.96$ KΩ. Actual implementation:

4.23



$\boxed{4.23}$ (a) C_1 implements $R_{eq1} = 1/C_1 f_{CK}$, and C_2 implements $R_{eq2} = 1/C_2 f_{CK}$. Consequently, $V_o = -\frac{1}{sR_{eq1}C_3}V_1 - \frac{1}{sR_{eq2}C_3}V_2 = -\frac{1}{s/\omega_1}V_1 - \frac{1}{s\omega_2}V_2$, $\omega_1 = (C_1/C_3)f_{CK}$, $\omega_2 = (C_2/C_3)f_{CK}$. Summing integrator.

(b) Flipping the switches to the left charges C_1 to $V_1 - V_2$, flipping the switches to the right dumps the charge $C(V_1-V_2)$ into the summing junction of the op amp. The average current is $I_{avg} = f_{CK}C_1(V_1-V_2)$, and $V_o = -\frac{1}{sC_2}I_{avg} = \frac{1}{s/\omega_0}(V_2-V_1)$, $\omega_0 = (C_1/C_2)f_{CK}$. Difference integrator.

(4.24)

4.24 (a) Denoting the current flowing into the summing junction of the op amp as I_{avg}, we have, by the superposition principle,

$$V_o = -\frac{1}{sC_2} I_{avg} = -\frac{1}{sC_2} f_{ck} C_1 (V_1 - V_2 + V_3)$$

$$= \frac{1}{s/\omega_0}(V_2 - V_1 - V_3), \quad \omega_0 = \frac{C_1}{C_2} f_{ck}.$$

(b) Switches flipped to ground $\Rightarrow C_1$ and C_2 are discharged. Switches flipped in the position shown $\Rightarrow C_1$ charges to V_i and C_2 to V_o. The current into the suming junction is thus $I_{avg} = f_{ck}(C_1 V_i + C_2 V_o)$, and $V_o = -(1/sC_3) I_{avg} = -(f_{ck}/sC_3)(C_1 V_i + C_2 V_o)$. Collecting terms,

$$H = \frac{V_o}{V_i} = -\frac{C_1}{C_2}\frac{1}{1 + s/\omega_0}, \quad \omega_0 = \frac{C_2}{C_3} f_{ck}.$$

4.25 (a) Let V_1 be the output of OA_1. Then, $V_o = -[(1/sC_2)/(1/sC_2)]V_i + [1/(s/\omega_0)]V_1 = -V_i + [1/(s/\omega_0)]V_1$, $\omega_0 = (C_0/C_2) f_{ck}$; $V_1 = -[(1/sC_2)/(1/sC_1)]V_o - [1/(s/\omega_0)](V_i + V_o) = -(C_1/C_2)V_o - [1/(s/\omega_0)](V_i + V_o)$. Eliminating V_1,

$$H = \frac{V_o}{V_i} = \frac{-[1 - (\omega/\omega_0)^2]}{1 - (\omega/\omega_0)^2 + (j\omega/\omega_0)/Q} = -H_N,$$

4.25

$Q = C_2/C_1$, $\omega_0 = (C_0/C_2) f_{ck}$.

(b) $C_0 = 1\,pF$, $C_2 = 15.9\,pF$, $C_1 = 1.59\,pF$.

4.26 (a) $C_2/C_1 = 250/(2\pi 2) = 19.9$; $C_1/C_3 = Q = 2/1$. Use $C_3 = 1\,pF$, $C_1 = 2\,pF$, $C_2 = 39.8\,pF$.

(b) $C_2/C_1 = 19.9$; $C_1/C_3 = 2/0.1 = 20$. Use $C_3 = 1\,pF$, $C_1 = 20\,pF$, $C_2 = 398\,pF$, quite large.

4.27 (a) $V_{BP} = \dfrac{1/j\omega C_2}{1/C_3 f_{clk}} V_{HP} = \dfrac{1}{j\omega/\omega_0} V_{HP}$,

$$\omega_0 = \frac{C_3}{C_2} f_{clk}.$$

$$V_{HP} = -\frac{1/j\omega C_2}{1/j\omega C_1} V_i - \frac{1/j\omega C_2}{1/j\omega C_1} V_{BP} - \frac{1/j\omega C_2}{1/j\omega C_3} V_{BP}$$

$$= -\frac{C_1}{C_2} V_i - \frac{C_1}{C_2} V_{BP} - \frac{1}{j\omega/\omega_0} V_{BP}.$$

Eliminating V_{HP},

$$V_{BP} = \frac{1}{j\omega/\omega_0}\left[-\frac{C_1}{C_2} V_i - \frac{C_1}{C_2} V_{BP} - \frac{1}{j\omega/\omega_0} V_{BP}\right].$$

Multiplying both sides by $(j\omega/\omega_0)^2$ and collecting,

$$V_{BP}\left[-(\omega/\omega_0)^2 + \frac{j}{Q}(\omega/\omega_0) + 1\right] = -\frac{j}{Q}(\omega/\omega_0) V_i, \quad Q = \frac{C_2}{C_1}.$$

4.26

$$\frac{V_{BP}}{V_i} = -\frac{(j\omega/\omega_0)/Q}{1-(\omega/\omega_0)^2+(j\omega/\omega_0)/Q} = -H_{BP}.$$

$$\frac{V_{HP}}{V_i} = \frac{V_{HP}}{V_{BP}}\frac{V_{BP}}{V_i} = j(\omega/\omega_0)\times(-H_{BP}) = -\frac{1}{Q}H_{HP}.$$

(b) $C_2/C_3 = 200/(2\pi 1) = 31.8$; $C_2/C_1 = 10$. Use $C_3 = 1\,pF$, $C_2 = 31.8\,pF$, $C_1 = 3.18\,pF$.

(c) $C_2/C_3 = 31.8$; $C_2/C_1 = 100$. Use $C_1 = 1\,pF$, $C_2 = 100\,pF$, $C_3 = 3.14\,pF$, a very reasonable spread considering the high Q value.

$\boxed{4.28}$ The normalized element values are $C_1 = C_5 = 1.14681$, $L_2 = L_4 = 1.37121$, $C_3 = 1.97500$. Using Eq. (4.29), $C_{C_1}/C_0 = 1.14681 \times 128 \times 10^3/(2\pi \times 3.4 \times 10^3) = 6.871$. $C_{L_2}/C_0 = 8.216$; $C_{C_3}/C_0 = 11.83$. A suitable set of capacitances is $C_0 = 1\,pF$, $C_{C_1} = C_{C_5} = 6.871\,pF$, $C_{L_2} = C_{L_4} = 8.216\,pF$, $C_{C_3} = 11.83\,pF$, $C_{R_i} = C_{R_o} = 1\,pF$.

(4.27)

$\boxed{4.29}$ (a) Let $f_1 = f_{CK}/100(50)$. Then, $V_{BP} = [1/(jf/f_1)]V_{HP}$ and $V_{LP} = [1/(jf/f_1)]V_{BP} = -[1/(f/f_1)^2]V_{HP}$. Using superposition,

$$V_{HP} = -\frac{R_2}{R_1}V_i - \frac{R_2}{R_4}\left[\frac{-1}{(f/f_1)^2}V_{HP}\right] - \frac{R_2}{R_3}\frac{1}{j(f/f_1)}V_{HP}.$$

Multiplying both sides by $\frac{R_4}{R_2}[j(f/f_1)]^2$,

$$V_{HP}\left[-\frac{R_4}{R_2}(f/f_1)^2 + 1 + \frac{R_4}{R_3}j(f/f_1)\right] = \frac{R_4}{R_1}(f/f_1)^2 V_i.$$

Let $\frac{R_4}{R_2}(f/f_1)^2 = (f/f_0)^2$, so that $f_0 = \sqrt{\frac{R_2}{R_4}}f_1 = \sqrt{\frac{R_2}{R_4}}\frac{f_{CK}}{100(50)}$. Then, $\frac{R_4}{R_3}j(f/f_1) = \frac{R_4}{R_3}\times \sqrt{\frac{R_2}{R_4}}\sqrt{\frac{R_4}{R_2}}j(f/f_1) = \frac{j}{Q}(f/f_0)$, $Q = \frac{R_3}{R_2}\sqrt{\frac{R_2}{R_4}}$.

Moreover, $\frac{R_4}{R_1}(f/f_1)^2 = \frac{R_2}{R_1}\frac{R_4}{R_2}(f/f_1)^2 = \frac{R_2}{R_1}(f/f_0)^2$. Thus,

$$\frac{V_{HP}}{V_i} = \frac{(R_2/R_1)(f/f_0)^2}{1-(f/f_0)^2 + (jf/f_0)/Q} = -\frac{R_2}{R_1}H_{HP}.$$

$$\frac{V_{BP}}{V_i} = \frac{V_{BP}}{V_{HP}}\frac{V_{HP}}{V_i} = \frac{1}{jf/f_1}\frac{V_{HP}}{V_i} =$$

$$\frac{\sqrt{R_4/R_2}}{\sqrt{\frac{R_4}{R_2}}(jf/f_1)}\frac{V_{HP}}{V_i} = \sqrt{\frac{R_4}{R_2}}\times\left(-\frac{R_2}{R_1}\right)\frac{1}{jf/f_0}H_{HP}$$

4.28

$$= -\frac{\sqrt{\frac{R_4}{R_2}}\frac{R_2}{R_3}\frac{R_3}{R_1}j(f/f_0)}{1-(f/f_0)^2+(jf/f_0)/Q} = -\frac{R_3}{R_1}\frac{(jf/f_0)/Q}{1-(f/f_0)^2+(jf/f_0)/Q}$$

$= -\frac{R_3}{R_1}H_{BP}$. Finally,

$$\frac{V_{LP}}{V_i} = \frac{V_{LP}}{V_{HP}}\frac{V_{HP}}{V_i} = \frac{-1}{(f/f_1)^2}\frac{V_{HP}}{V_i} = \frac{-R_4/R_2}{(R_4/R_2)(f/f_1)^2}\frac{V_{HP}}{V_i}$$

$$= -\frac{R_4}{R_2}\frac{1}{(f/f_0)^2}\frac{R_2}{R_1}\frac{(f/f_0)^2}{1-(f/f_0)^2+(jf/f_0)/Q}$$

$$= -\frac{R_4}{R_1}H_{LP}.$$

(b) $V_N = -\frac{R_G}{R_L}V_{LP} - \frac{R_G}{R_H}V_{HP}$. Eq. (4.34):

$$H_N = \frac{V_N}{V_i} = -\frac{R_G}{R_L}H_{0LP}H_{LP} - \frac{R_G}{R_H}H_{0HP}H_{HP}.$$

$$\lim_{f\to 0} H_N = -\frac{R_G}{R_L}H_{0LP} = \frac{R_G}{R_L}\frac{R_4}{R_1}.$$

$$\lim_{f\to f_{clk}/2} H_N = -\frac{R_G}{R_H}H_{0HP} = \frac{R_G}{R_H}\frac{R_2}{R_1}.$$ Moreover,

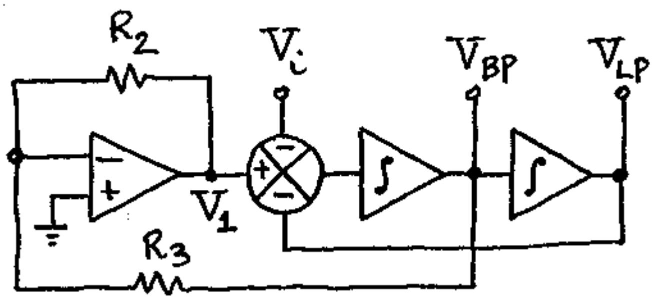
$$H_N = -\frac{R_G}{R_L}H_{0LP}\frac{1+\frac{R_L}{R_G}\frac{R_G}{R_H}\frac{H_{0HP}}{H_{0LP}}\left[-(f/f_0)^2\right]}{1-(f/f_0)^2+(jf/f_0)/Q}$$

$$= -\frac{R_G}{R_L}H_{0LP}\frac{1-(f/f_z)^2}{1-(f/f_0)^2+(jf/f_0)/Q},$$ where

$$\frac{1}{f_z^2} = \frac{R_L}{R_H}\frac{R_2}{R_4}\frac{1}{f_0^2} = \frac{R_L}{R_4}\frac{1}{f_x^2} \Rightarrow f_z = \sqrt{\frac{R_H}{R_L}}\frac{f_{ck}}{100(50)}.$$

4.29

4.30 (a) $V_1 = -(R_2/R_3)V_{BP}$. $V_{BP} = \frac{1}{jf/f_0} \times (V_1 - V_i - V_{LP})$



$V_{LP} = \frac{1}{jf/f_0} V_{BP}$.

Substituting,

$$V_{BP} = \frac{1}{jf/f_0}\left[-\frac{R_2}{R_3}V_{BP} - V_i - \frac{1}{jf/f_0}V_{BP}\right].$$

Multiply both sides by $[j(f/f_0)]^2$. Then,

$$V_{BP}\left[-(f/f_0)^2 + (jf/f_0)/Q + 1\right] = -(jf/f_0)V_i, \; Q = \frac{R_3}{R_2}.$$

$$\frac{V_{BP}}{V_i} = \frac{-Q \times (jf/f_0)/Q}{1-(f/f_0)^2 + (jf/f_0)/Q} = -QH_{BP}.$$

$$\frac{V_{LP}}{V_i} = \frac{V_{LP}}{V_{BP}}\frac{V_{BP}}{V_i} = \frac{1}{jf/f_0} \times (-QH_{BP}) = -H_{LP}.$$

(b) Tie the 50/100/CL pin to ground to make $f_0 = f_{clk}/100$. Then, $f_{clk} = 100 \times 500 = 50$ kHz, $R_3/R_2 = Q = 10$. Use $R_2 = 10\ \text{k}\Omega$, $R_3 = 100\ \text{k}\Omega$.

4.31 $V_N = -\frac{R_2}{R_1}V_i - \frac{R_2}{R_3}V_{BP} - \frac{R_2}{R_4}V_{LP}$;

$V_{LP} = \frac{1}{jf/f_1}V_{BP}$; $V_{BP} = \frac{1}{jf/f_1}(V_N - V_{LP})$,

$f_1 = \frac{f_{CK}}{100(50)}$. Eliminating V_{LP} and V_{BP}, collecting and solving for the ratio V_N/V_i,

$$\frac{V_N}{V_i} = \frac{-\frac{R_2}{R_1}\frac{R_4}{R_2+R_4}\left[1-(f/f_1)^2\right]}{1-\frac{R_4}{R_2+R_4}(f/f_1)^2+\frac{R_2}{R_3}\frac{R_4}{R_2+R_4}(jf/f_1)}.$$

Letting $f_z = \frac{f_{CK}}{100(50)}$, $f_0 = f_z\sqrt{1+R_2/R_4}$, $Q = \frac{R_3}{R_2}\sqrt{1+\frac{R_2}{R_4}}$ gives $\frac{V_N}{V_i} = H_{0N}H_N$, $H_{0N} = \frac{-R_2/R_1}{1+R_2/R_4}$

4.32 $V_{AP} = -\frac{R_2}{R_1}V_i - \frac{R_2}{R_3}V_{BP}$; $V_{BP} = \frac{1}{jf/f_0}(V_{AP} - V_i - V_{LP})$; $V_{LP} = \frac{1}{jf/f_0}V_{BP}$,

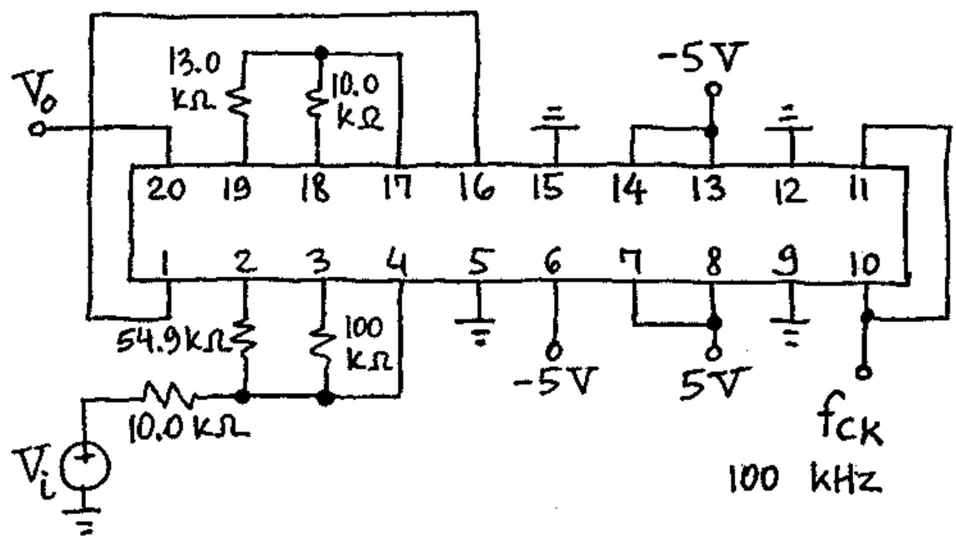
$f_0 = \frac{f_{CK}}{100(50)}$. Eliminating V_{BP} and V_{LP}, collecting, and solving for the ratio V_{AP}/V_i,

$$\frac{V_{AP}}{V_i} = -\frac{R_3/R_1}{R_3/R_2}\,\frac{1-(f/f_0)^2+(R_1/R_3)(jf/f_0)}{1-(f/f_0)^2+(R_2/R_3)(jf/f_0)}.$$

By inspection, $H_{0AP} = -R_2/R_1$, $f_0 = f_{CK}/[100(50)]$, $Q = R_3/R_2$, $Q_z = R_3/R_1$.

(4.31)

4.33 Table 4.1 indicates that we need two stages with $Q_1 = 0.5418$ and $Q_2 = 1.3065$, respectively. Let the first stage be as in Fig. 4.36. Then, $R_3/R_2 = 0.542$ and $R_2/R_1 = 10$. Use $R_1 = 10.0\ k\Omega$, $R_2 = 100\ k\Omega$, and $R_3 = 54.9\ k\Omega$, all 1%. Let the second stage be as in Problem 4.30. Then, $R_3/R_2 = 1.3065$. Use $R_2 = 10.0\ k\Omega$, $R_3 = 13.0\ k\Omega$, 1%.



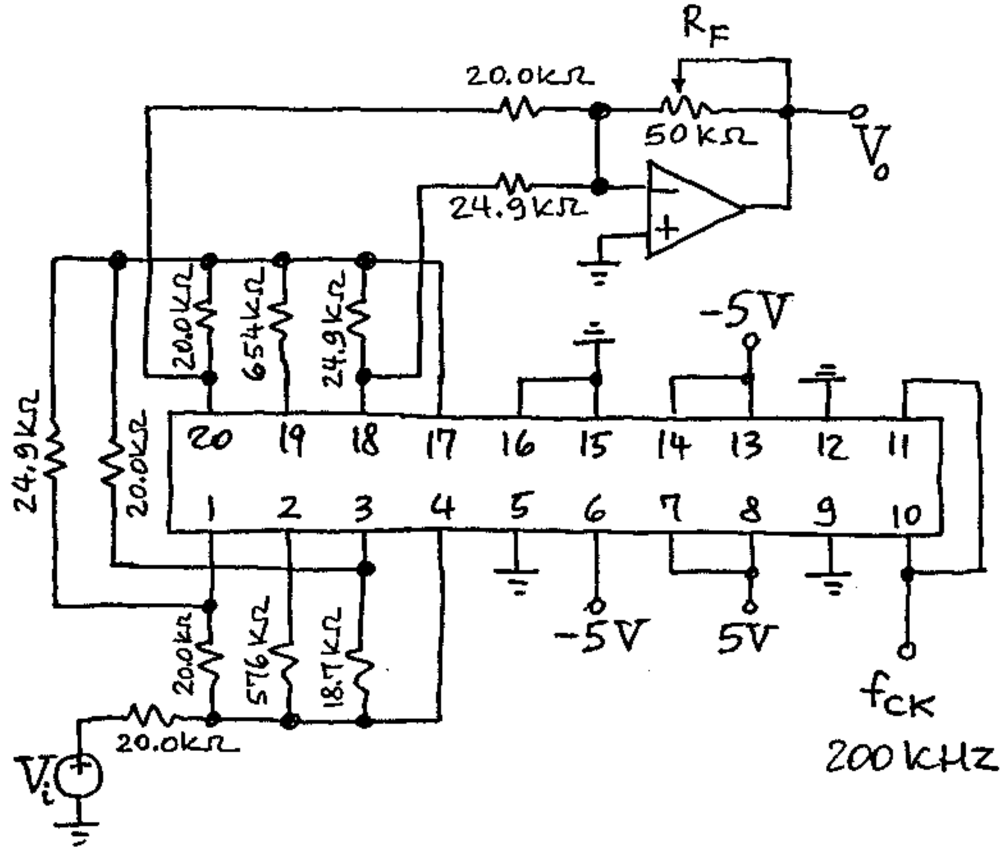
4.34 Use two configurations of the type of Fig. 4.37. 1st stage: $R_1 = R_3 = 20.0\ k\Omega$, $R_2 = 5.36\ k\Omega$, $R_4 = 8.87\ k\Omega$. 2d stage: $R_1 = 20.0\ k\Omega$, $R_2 = 28.0\ k\Omega$, $R_3 = 63.4\ k\Omega$, $R_4 = 16.9\ k\Omega$. Impose $f_0 = f_{CK}/100$, $f_{CK} = 200$ kHz.

4.35 Use the configuration of Fig. 4.40. 1st stage: $R_1 = 392\,k\Omega$, $R_2 = 19.6\,k\Omega$, $R_3 = 374\,k\Omega$, $R_4 = 17.8\,k\Omega$, $R_H = 10.0\,k\Omega$, $R_L = 10.0\,k\Omega$. 2d stage: $R_2 = 20.0\,k\Omega$, $R_3 = 422\,k\Omega$, $R_4 = 21.5\,k\Omega$, $R_H = 20.0\,k\Omega$, $R_L = 20.0\,k\Omega$, $R_G = 20.0\,k\Omega$.

4.36 Stage 1: Let $R_1 = R_4 = 20.0\,k\Omega$. Then, $R_2/R_4 = (1.948/2)^2 \Rightarrow R_2 = 18.97\,k\Omega$ (use $18.7\,k\Omega$). $R_3 = Q\sqrt{R_2 R_4} = 574.3\,k\Omega$ (use $576\,k\Omega$). Let $R_{HA} = 20.0\,k\Omega$. Then, $R_{LA} = R_{HA}/(1.802/2)^2 = 24.6\,k\Omega$ (use $24.9\,k\Omega$).

Stage 2: Let $R_4 = 20.0\,k\Omega$. Then, $R_2 = 24.9\,k\Omega$, $R_3 = 654\,k\Omega$, $R_{LA} = 20.0\,k\Omega$, $R_{HA} = 24.9\,k\Omega$. R_F controls the resonance gain.

4.33



4.37 From Table 4.1,

$f_{01} = 500/1.034 = 483.56\text{ Hz}$, $Q_1 = 8.082$

$f_{02} = 500/0.894 = 559.28\text{ Hz}$, $Q_2 = 2.453$

$f_{03} = 500/0.645 = 775.19\text{ Hz}$, $Q_3 = 1.183$

$f_{04} = 500/0.382 = 1308.9\text{ Hz}$, $Q_4 = 0.593$

Stage 1: $\sqrt{R_2/R_4} = 1/1.034 \Rightarrow$ $R_2/R_4 = 1/1.034^2 = 0.935$; $R_3/R_2 = Q/\sqrt{R_2/R_4} = 8.082 \times 1.034 = 8.357$; $R_2/R_1 = H_{0HP} = 1$. Let $R_1 = R_2 = 20.0\text{ k}\Omega$. Then, $R_3 = 8.357 \times 20 = 167.14\text{ k}$ (use $169\text{ k}\Omega$) and $R_4 = 20/0.935 = 21.4\text{ k}\Omega$ (use $21.5\text{ k}\Omega$).

(4.34)

Stage 2: $R_2/R_4 = 1.251$; $R_3/R_2 = 2.193$; $R_2/R_1 = 1$. Use $R_1 = R_2 = 20.0\ k\Omega$, $R_3 = 44.2\ k\Omega$, $R_4 = 15.8\ k\Omega$.

Stage 3: $R_2/R_4 = 2.40$; $R_3/R_2 = 0.763$; $R_2/R_1 = 1$. Use $R_1 = R_2 = 20\ k\Omega$, $R_3 = 15.4\ k\Omega$, $R_4 = 8.45\ k\Omega$.

Stage 4: $R_2/R_4 = 6.853$; $R_3/R_2 = 0.2265$; $R_2/R_1 = 1$. Use $R_1 = R_2 = 20.0\ k\Omega$, $R_3 = 4.53\ k\Omega$, $R_4 = 2.94\ k\Omega$.