(6.1)

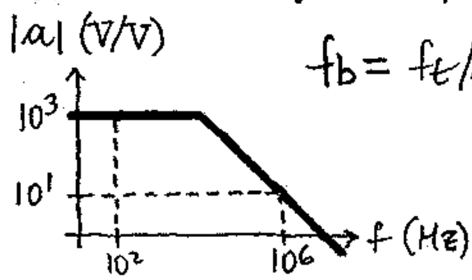
6.1 (a) By Eq. (6.1), $a_0 = k_1 \times a_2$; by Eq. (6.9), f_t is independent of a_2; by Eq. (6.5), $f_b = k_2/a_2$. Thus, a $\pm 20\%$ variation of a_2 causes a $\pm 20\%$ variation of a_0 and a variation of about $\mp 20\%$ of f_b.

(b) By Eq. (6.9), $f_t = k_3/C_c$; by Eq. (6.1), a_0 is independent of C_c; by Eq. (6.5), $f_b = k_4/C_c$. Thus, a $\pm 10\%$ variation of C_c causes variations of about $\mp 10\%$ in both f_b and f_t.

6.2 $-\tan^{-1}(80/f_a) = -58^\circ \Rightarrow f_b = 50$ Hz; Since 1 Hz << 50 Hz, $a_0 \cong |a(j1\text{Hz})| = 10^5$ V/V; $f_t = a_0 f_b = 5$ MHz.

6.3 $a_0 = 10^3$ V/V; $f_t = 10^6 \times 10 = 10^7$ Hz

$f_b = f_t/a_0 = 10^4$ Hz.



(a) $\tan^{-1}(f/10^4) = 60^\circ$

$\Rightarrow f = 17.32$ kHz

(b) $10^3/[1+(f/10^4)^2]^{\frac{1}{2}} = 2$

$\Rightarrow f = 5$ MHz.

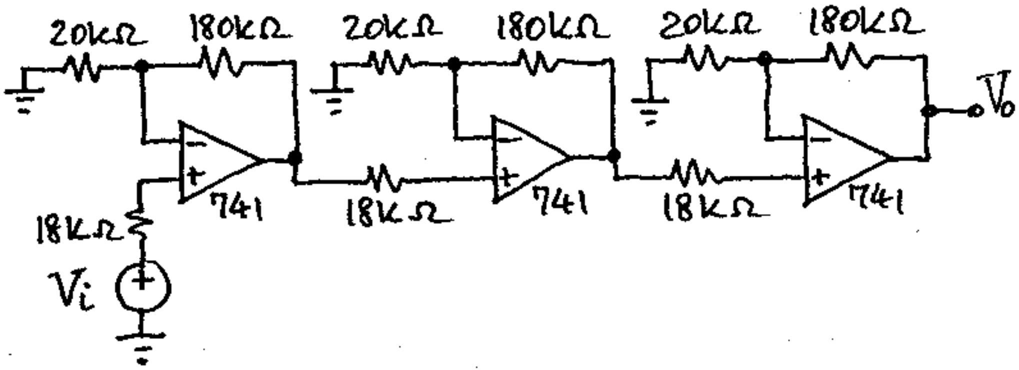
6.4 $A = \left(\dfrac{A_1}{1+jf/f_{B1}}\right)^2 = \dfrac{A_1^2}{1-(f/f_{B1})^2 + 2jf/f_{B1}} = H_{0LP} H_{LP}$,

$H_{0LP} = A_1^2 = 10^3$ V/V, $f_{B1} = 31.6$ kHz, $Q = 1/2$.

6.5 (a) Impose $\{A_0/[1+(f/f_{B0})^2]^{1/2}\}^n = A_0^n/2^{1/2}$
$\Rightarrow [1+(f/f_{B0})^2]^n = 2 \Rightarrow f = f_{B0}\sqrt{2^{1/n}-1}$, $f_{B0} = f_t/A_0$.

(b) The same expression holds, but with f_t/A_0 replaced by $f_t/(A_0+1)$.

6.6 $A_1 = A_2 = A_3 = 10/[1+j(f/100\text{kHz})]$.



$A = A_1^3 \Rightarrow |A| = 1{,}000/[1+(f/10^5)^2]^3$. We wish to find the frequency f_B at which $|A| = 1000/\sqrt{2}$, that is,

$$\frac{1{,}000}{\left[\sqrt{1+(f_B/10^5)^2}\,\right]^3} = \frac{1{,}000}{\sqrt{2}}.$$

$f_B = 10^5\sqrt{2^{1/3}-1} = 51\text{kHz}$. As expected, decreasing the individual gains increases the overall bandwidth.

6.7 (a) $A = \dfrac{2}{1+jf/(5/2\ \text{MHz})} \times \dfrac{-2}{1+jf/(5/3\ \text{MHz})}$.

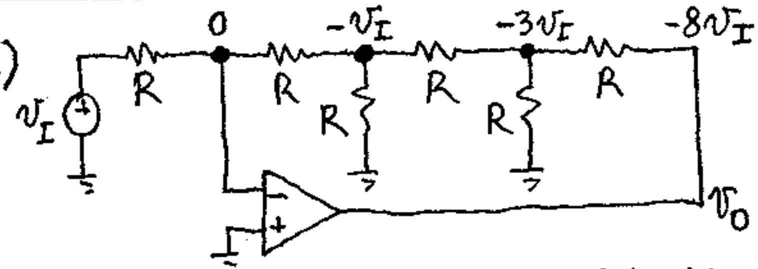
Imposing $[1+(f/2.5)^2]^{1/2} \times [1+(f/1.\overline{6})^2]^{1/2} = 2^{1/2}$ gives $f_{-3dB} = 1.276$ MHz.

(b) Imposing $|A| = 0.99 \times 4$ gives $f_{-1\%} =$

(6.3)

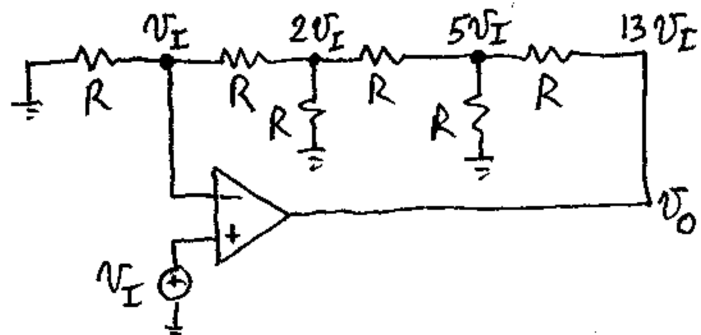
196 kHz. Imposing $\tan^{-1}(f/2.5) + \tan^{-1}(f/1.\bar{6}) = 5°$ gives $f_{-5°} = 87.3$ kHz.

6.8 (a)



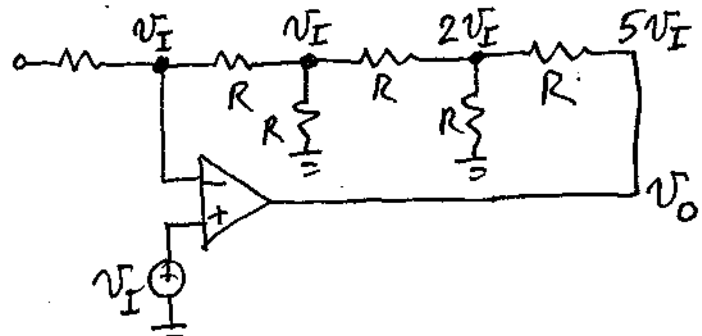
$A_0 = -8$ V/V; $\beta = \frac{R}{R+R} \times \frac{R\|2R}{R+(R\|2R)} \times \frac{R\|[R+(R\|2R)]}{R+R\|[R+(R\|2R)]} =$ $\frac{1}{13}$ V/V; $f_B = \beta f_t = \frac{4}{13}$ MHz; $GBP = |A_0| f_B = \frac{32}{13}$ MHz.

(b)



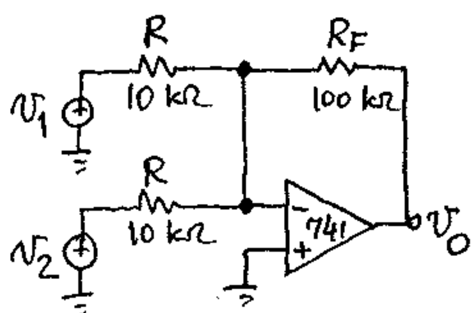
$A_0 = 13$ V/V; $\beta = 1/13$ V/V; $GBP = A_0 \beta f_t = 4$ MHz.

(c)



$A_0 = 1/\beta = 5$ V/V; $GBP = 4$ MHz.

6.9 (a)



$\beta = \frac{R/2}{R/2 + R_F}$

$= 1/21$ V/V;

$f_B = \beta f_t$

$= 47.6$ kHz.

(b) With five inputs instead of two, we get $\beta = (R/5)/[R/5 + R_F] = 1/51$ V/V, so $f_B = 19.6$ kHz. Increasing the number of inputs decreases β and, hence, the -3 dB frequency.

6.10 $f_{-3dB} = \beta f_t = \beta 10^6$ Hz.

Fig. P1.17: $\beta = 50/(50+20) - 10/(10+40) = \frac{18}{35}$ V/V; $f_{-3dB} = 514$ kHz.

Fig. P1.19: $\beta = \frac{3+2+1}{4+3+2+1} - \frac{1}{1+2+3+4} = 0.5$ V/V; $f_{-3dB} = 500$ kHz.

Fig. P1.21: $\beta = R_1/(R_1+R_2) = 0.5$ V/V, regardless of the switch position; $f_{-3dB} = 500$ kHz.

Fig. P1.61: $\beta = 10/(10+30) = 1/4$ V/V; $f_{-3dB} = 250$ kHz.

6.11 $\beta_{II} = (1 + R_2/R_1)^{-1} = 2/3$ V/V; $f_{II} = \beta_{II} f_t = 5.\bar{3}$ MHz. $\beta_I = (1 + 2R_3/R_G)^{-1}$, $50\,\Omega \le R_G \le 100.05$ kΩ; $1/2000$ V/V $\le \beta_I \le 1/2$ V/V; 4 kHz $\le f_I \le 4$ MHz.

Wiper up: $$H = \frac{V_o}{V_2 - V_1} = \frac{2000}{1 + jf/(4\text{kHz})} \times \frac{0.5}{1 + jf/(5.\bar{3}\text{MHz})}$$

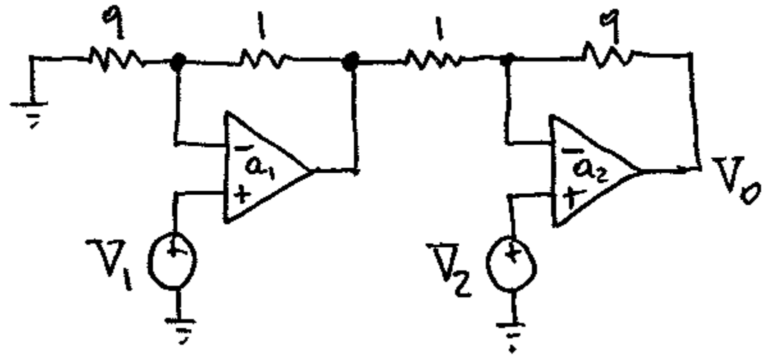
First stage dominates, so $f_{-3dB} \cong 4$ kHz.

Wiper down: $$H = \frac{1}{1 + jf/(4\text{MHz})} \times \frac{0.5}{1 + jf/(5.\bar{3}\text{ MHz})}$$

Impose $\left\{\left[1 + \left(\frac{f}{4\text{MHz}}\right)^2\right]\left[1 + \left(\frac{f}{5.\bar{3}\text{MHz}}\right)^2\right]\right\}^{1/2} = \sqrt{2}$ gives $f_{-3dB} = 2.93$ MHz.

(6.5)

6.12



$\beta_1 = 0.9$ V/V ; $f_1 = 900$ kHz; $\beta_2 = 0.1$; $f_2 = 100$ kHz.

$$V_o = \frac{10}{1+jf/10^5}V_2 - \frac{9}{1+jf/10^5}\,\frac{1/0.9}{1+jf/(900\times10^3)}V_1$$

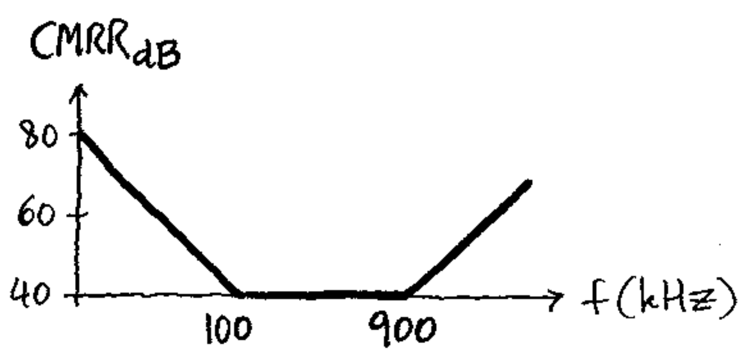
$$= \frac{10}{1+jf/10^5}\left[V_2 - \frac{1}{1+jf/(0.9\times10^5)}V_1\right]$$

Clearly, V_2 is processed with $f_{-3dB} = 100$ kHz. To find that of V_1, impose $\sqrt{1+(f/10^5)^2} \times \sqrt{1+[f/(0.9\times10^5)]^2} = \sqrt{2}$. Then, $f_{-3dB} = 61$ kHz.

6.13 Let $V_1 = V_2 = V_{cm}$. From Prob. 6.12:

$$V_o = \frac{A_{dm}}{1+jf/10^5}\left[1 - \frac{1}{1+jf/(0.9\times10^5)}\right]V_{cm} = A_{cm}V_{cm}$$

$$CMRR = \frac{A_{dm}}{A_{cm}} = \frac{[1+jf/10^5]\times[1+jf/(0.9\times10^5)]}{jf/(0.9\times10^5)}$$



(6.6)

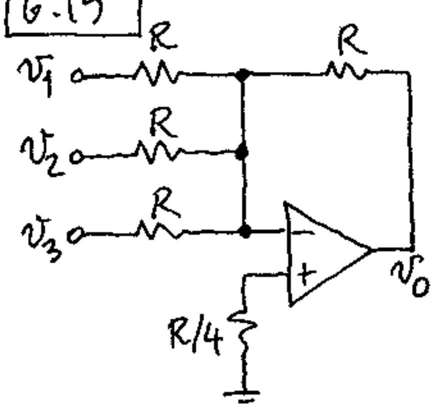
6.14 By the superposition principle,

$E_o = 10\,(V_{OS2} - V_{OS1}) + (1 + A_{II})V_{OS3}$;

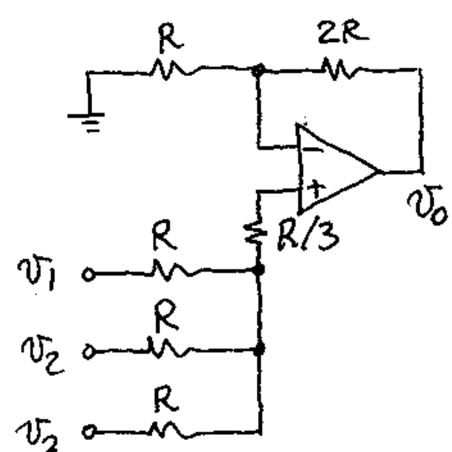
$E_{o(max)} = (21 + A_{II})V_{OS}$. Thus, to minimize $E_{o(max)}$ one should specify A_{II} as small as possible. For instance, letting $A_I = 10$ V/V and $A_{II} = 1$ V/V gives $E_{o(max)} = 22\,V_{OS}$.

We have $\beta_I = 1/A_I$ and $\beta_{II} = 1/(A_{II}+1)$. So, $f_I = \beta_I f_t$ and $f_{II} = \beta_{II} f_t$. The overall bandwidth is maximized when $f_I = f_{II}$, i.e. when $A_I = A_{II}+1$; but, $A_I \times A_{II} = 10$ V/V, so $(A_{II}+1)A_{II} = 10 \Rightarrow A_{II} = 2.7$ V/V and $A_I = 3.7$ V/V. In this case we get $E_{o(max)} = 23.7\,V_{OS}$.

6.15



(a)



(b)

(a): $\beta = (R/3)/(R/3 + R) = 1/4$ V/V; $E_o = 4E_I$; $f_B = f_t/4$.

(b): $\beta = R/(R+2R) = 1/3$ V/V; $E_o = 3E_I$; $f_B = f_t/3$; (b) is preferable in both cases.

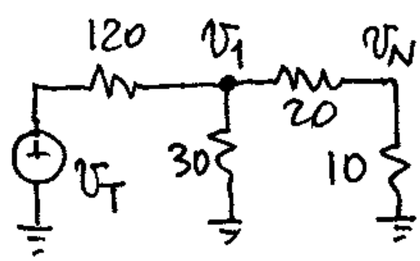
6.16 (a) $f_{-3dB} = f_t$, $R_i = \infty$, $R_o = 0$, least No. of components; lack of flexibility.

(b) $f_{-3dB} = f_t/2$, $R_i = \infty$, $R_o \neq 0$, more complex; gain can be altered by changing the resistors.

(c) $f_{-3dB} = (f_t/2)\sqrt{\sqrt{2}-1} = f_t/3.1$, $R_i \neq \infty$, $R_o = 0$; most complex, most flexible.

6.17 Large open-loop dc gain implies $I_{2k\Omega} \to 0$; consequently, $I_{1k\Omega} = I_{3k\Omega} \to 0$, and $V_o = (1 + 32/16)V_i = 4V_i \Rightarrow A_0 = 4$ V/V. From Prob. 1.60, $\beta = 1/30$; $f_B = 3\times10^6/30 = 100$ kHz.

6.18



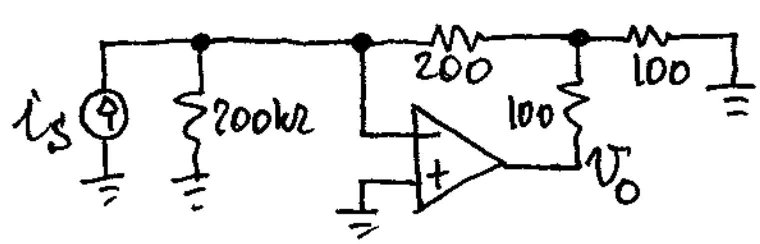
$$v_N = \frac{10}{20+10} v_1$$

$$= \frac{1}{3} \frac{30 /\!/ 130}{120 + (30 /\!/ 130)} v_T$$

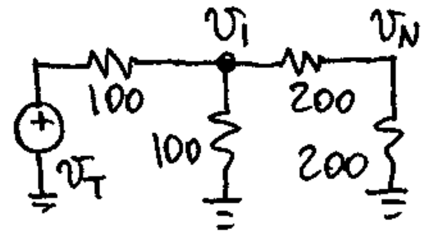
$$\beta = v_N / v_T = 1/27.$$

$f_B = \frac{27}{27} = 1$ MHz; $A_0 = -\frac{20}{10}\left(1 + \frac{120}{20} + \frac{120}{30}\right) = -22$ V/V. GBP = 22 MHz.

6.19 $f_t = 10^{80/20} \times 1.8 \times 1.8 \times 10^3 = 18$ MHz.



(6.8) $v_O = -200\left(1 + \frac{100}{200} + \frac{100}{100}\right) i_S \Rightarrow A_0 = -0.5\,V/\mu A.$

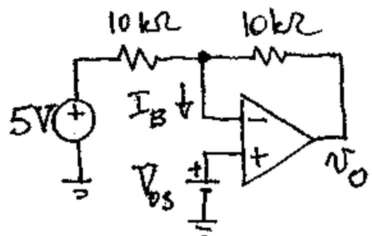


$v_N = \frac{1}{2} v_1$

$= \frac{1}{2} \frac{100 // 400}{100 + (100 // 400)} v_T$

$\beta = v_N / v_T = 2/9$; $f_B = (2/9)18 = 4$ MHz.

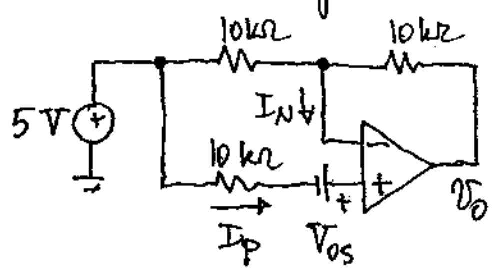
$\boxed{6.20}$ $f_{-3dB} = \beta f_t = 0.5 f_t = 500$ kHz regardless of the switch position. <u>Switch closed:</u>



$E_{I(max)} = V_{OS0} + |v_O|/a_o + 0.5RI_B = 0.75 \times 10^{-3} + 5/50{,}000 + 0.5 \times 10^4 \times 50 \times 10^{-9} = 1.1$ mV;

$E_{o(max)} = (1/\beta) E_{I(max)} = 2.2$ mV $\Rightarrow v_O = -4.9978$ V.

Switch open:



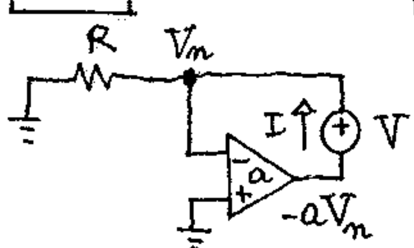
$E_I = \pm V_{OS0} \pm 5/CMRR - 5/a_0 + 0.5RI_N - RI_P$

$= \pm V_{OS0} \pm 5/CMRR - 5/a_0 - 0.5RI_B$

$E_{I(max)} = -[0.75 \times 10^{-3} + 5/10^{100/20} + 5/50{,}000 + 0.5 \times 10^4 \times 50 \times 10^{-9} = 1.15$ mV;

$E_{O(max)} = 2.3$ mV.

$\boxed{6.21}$

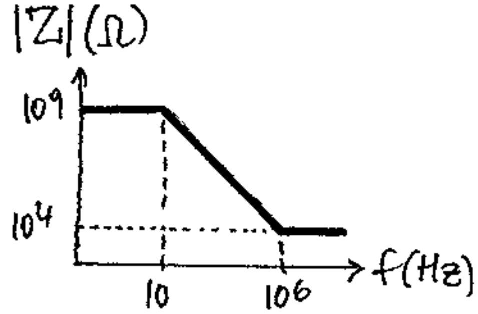


KVL: $V = V_n - (-aV_n) = (1+a)V_n$

$\Rightarrow V_n = V/(1+a)$

$Z_o = \frac{V}{I} = \frac{V}{V_n/R} = R(1+a)$

(6.9)



$$Z_o = R\left[1 + \frac{a_o}{1+jf/f_a}\right]$$

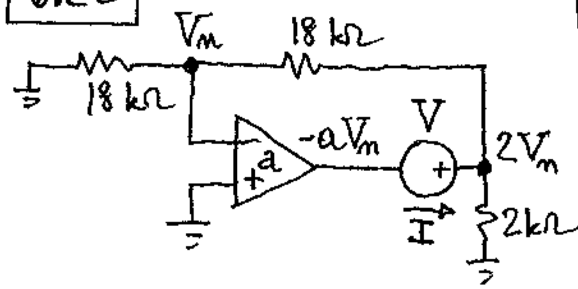
$$\cong R(1+a_o)\frac{1+jf/f_t}{1+jf/f_a}$$

$$\cong 10^9 \frac{1+jf/10^6}{1+jf/10}\ \Omega.$$

Capacitive impedance:

$R_p = 10^9\ \Omega$, $R_s = 10^4\ \Omega$, $C_{eq} = 1/2\pi \times 10^9 \times 10 = 15.9$ pF.

6.22



KVL: $V = 2V_n - (-aV_n)$

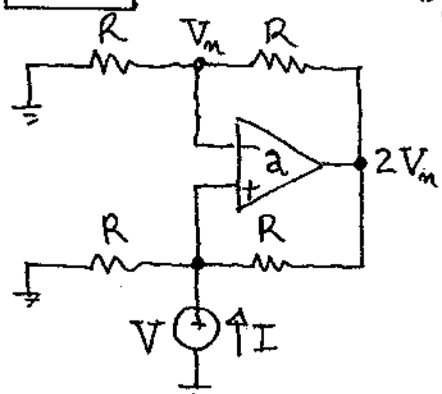
$\Rightarrow V_n = V/(2+a)$;

KCL: $I = 2V_n \times \left(\frac{1}{2\text{ k}\Omega} + \frac{1}{36\text{ k}\Omega}\right)$;

$$I = \frac{19}{18\text{ k}\Omega}\frac{V}{2+a}\ ;\ Z_o = \frac{V}{I} = \frac{18\text{ k}\Omega}{19}\left(2 + \frac{10^5}{1+jf/10}\right)$$

$$Z_o = \frac{18}{19}10^8 \frac{1+jf/(500\text{ kHz})}{1+jf/(10\text{ Hz})}\ \Omega;\ \text{capacitive.}$$

6.23



By op amp action,

$2V_n = a(V - V_n) \Rightarrow$

$V_n = Va/(2+a)$; KCL:

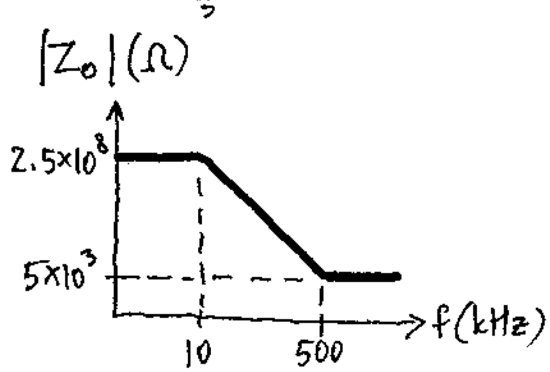
$$I = \frac{V}{R} + \frac{V-2V_n}{R} = \frac{2V}{R(1+a/2)}$$

$$Z_o = \frac{V}{I} = \frac{R}{2}\left(1+\frac{a}{2}\right)$$

$$= (5\text{ k}\Omega)\left(1+\frac{10^5/2}{1+jf/10}\right)$$

$$\cong 2.5\times10^8 \frac{1+jf/(500\text{ kHz})}{1+jf/10}\ \Omega$$

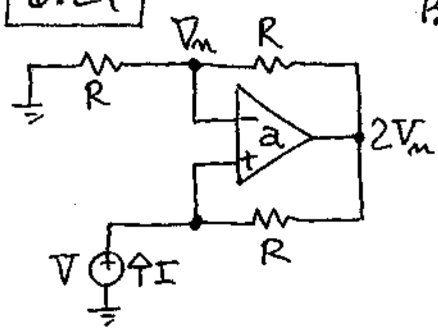
$\Rightarrow$ Capacitive impedance.



(6.10)

At low frequencies, where a is high, Z_o is also high (ideally, $Z_o \to \infty$ for $a \to \infty$). As gain a starts to roll off, so does Z_o. At high frequencies, where $a \to 0$ and the op amp output thus behaves as a 0-V source, we have: $Z_o \to R // R = 5\ \mathrm{k\Omega}$.

6.24



By op amp action,

$2V_n = a(V - V_n) \Rightarrow$

$V_n = Va/(1+a)$; Ω Law:

$I = (V - 2V_n)/R$

$= (V/R)[1 - 2a/(1+a)]$

$$Z_{eq} = \frac{V}{I} = R\frac{1+a/2}{1-a/2} = R\frac{1+a_0/2}{1-a_0/2}\ \frac{1+jf/[f_a(1+a_0/2)]}{1-jf/[f_a(1+a_0/2)]}.$$

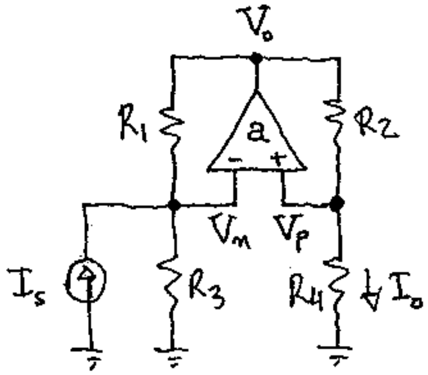
Ignoring 1 compared to $a_0/2$, and letting $f_b a_0 = f_t$ gives

$$Z_{eq} = -R\frac{1+jf/(f_t/2)}{1-jf/(f_t/2)} = -10^4\frac{1+jf/(500\ \mathrm{kHz})}{1-jf/(500\ \mathrm{kHz})}\ \Omega.$$

As f is varied from 0 to ∞, Z_{eq} changes from $-10\ \mathrm{k\Omega}$ to $+10\ \mathrm{k\Omega}$; moreover, $|Z_{eq}| = 10\ \mathrm{k\Omega}$ = constant regardless of frequency.

6.11

6.25 Circuit to find the gain $A = I_o/I_s$:



$V_o = (R_2 + R_4) I_o$; $V_p = R_4 I_o$;

$V_o = a(V_p - V_n) \Rightarrow V_n = V_p - V_o/a$

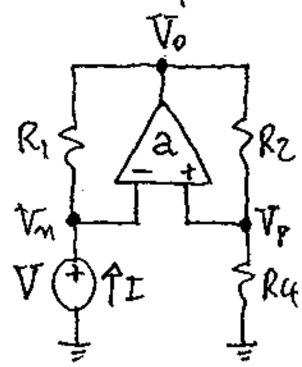
$\Rightarrow V_n = \left(R_4 - \dfrac{R_2 + R_4}{a}\right) I_o$; KCL:

$$I_s + \frac{0 - V_n}{R_3} + \frac{V_o - V_n}{R_1} = 0$$

Eliminating V_n, collecting, and substituting the resistance values gives:

$$A = \frac{I_o}{I_s} = \frac{-15/2}{7 + 12/a} = \frac{-7.5}{7 + 12(1 + jf/f_b)/a_0} \cong \frac{-(15/14)\ \text{A/A}}{1 + jf/(7f_t/12)} .$$

Circuit to find the input impedance Z_i:



$V_p = \dfrac{R_4}{R_2 + R_4} V_o = \dfrac{1}{6} V_o$

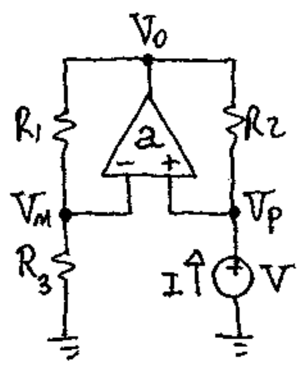
$V_o = a(V_p - V_n) = a\left(\dfrac{1}{6} V_o - V\right)$

$\Rightarrow V_o = 6aV/(a - 6)$

$I = \dfrac{V - V_o}{R_1} = \dfrac{1}{R_1}\left(V - \dfrac{6a}{a - 6} V\right)$

$$Z_i = \frac{V}{I} = (-2\ \text{k}\Omega)\frac{a - 6}{a + 1.2} \cong (-2\ \text{k}\Omega) \times \frac{1 - jf/(f_t/6)}{1 + jf/(f_t/1.2)} .$$

Circuit to find the output impedance Z_o:



$V_n = \dfrac{R_3}{R_1 + R_3} V_o = 0.75 V_o$

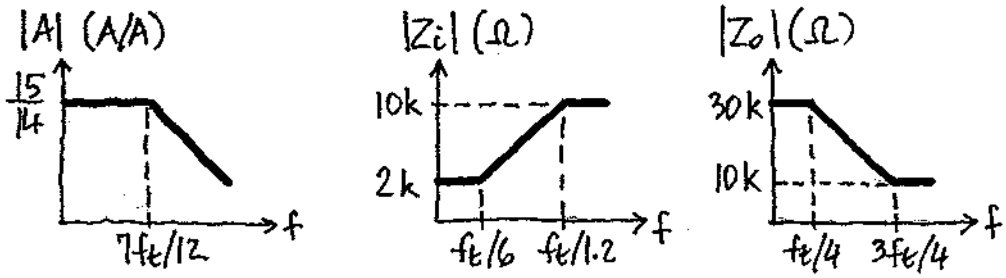
$V_o = a(V_p - V_n) = a(V - 0.75 V_o)$

$\Rightarrow V_o = aV/(1 + 0.75a)$

$I = \dfrac{V - V_o}{R_2} = \dfrac{1}{R_2}\left(V - \dfrac{a}{1 + 0.75a} V\right)$

6.12

$$Z_o = \frac{V}{I} = (-30\,k\Omega)\frac{a+4/3}{a-4} = (-30\,k\Omega)\frac{1+jf/(0.75f_t)}{1-jf/((0.25f_t)}$$

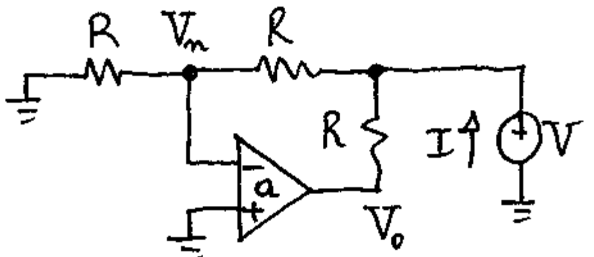


6.26 $\beta = 10/(10+20) = 1/3$ V/V.

$Z_o(f\to 0) = \dfrac{r_o}{1+a_0\beta_0} = \dfrac{100}{1+10^5/3} = 3\,m\Omega.$

$f_b = f_t/a_0 = 40$ Hz. $L_{eq} = Z_o(f\to 0)/2\pi f_b = 3\times 10^{-3}/2\pi 40 = 11.9\,\mu H$. $f_{res} = 1/2\pi\sqrt{LC} = 146$ kHz. $Q = 1/\sqrt{(3\times 10^{-3})\times\sqrt{C/L}} = 3641.$

6.27 $f_t = 300\times 10^3\times 10 = 3$ MHz. Test voltage:

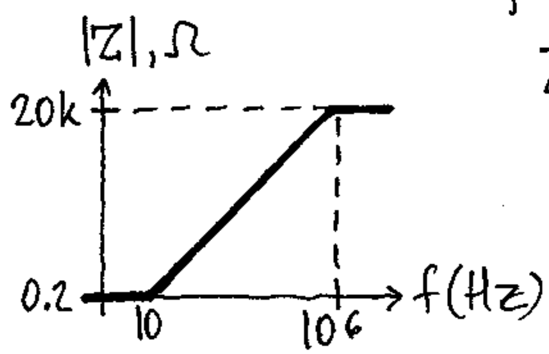


$V_n = (1/2)V$;

$V_o = -aV_n = -\frac{a}{2}V$;

$I = \dfrac{V}{2R} + \dfrac{V-V_o}{R}$.

Eliminating V_o gives $Z = \dfrac{V}{I} = \dfrac{2R}{3+a}$. Letting $a = 300{,}000/[1+jf/10]$ finally gives

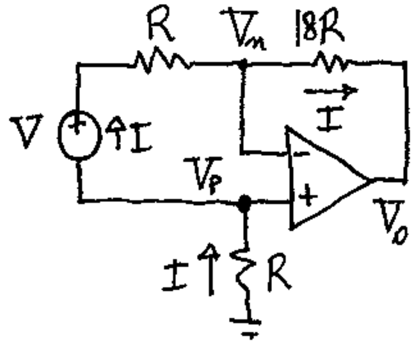


$$Z = \frac{2R}{a_0}\,\frac{1+jf/f_b}{1+jf/f_t}$$

$$= 0.2\Omega\,\frac{1+jf/10}{1+jf/10^6}.$$

(6.13)

6.28



$V_p = -RI$; $V_n = V - 2RI$

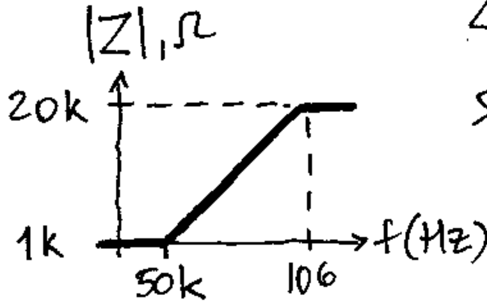
$V_o = a(V_p - V_n) = a(RI - V)$

$I = \dfrac{V_n - V_o}{18R} = \dfrac{V(1+a) - RI(2+a)}{18R}$

Collecting,

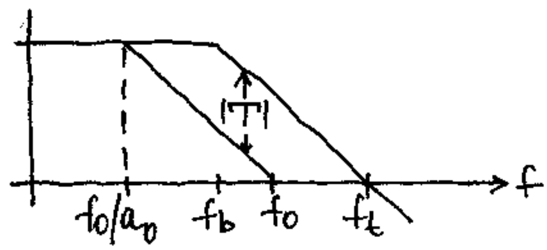
$Z = \dfrac{V}{I} = R\dfrac{20+a}{1+a}$. Substituting $a \cong 1/(jf/f_t)$,

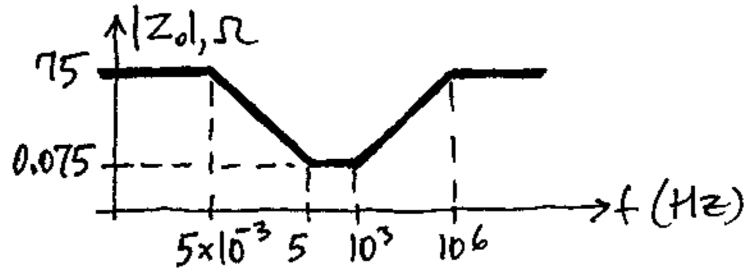
$Z = 1\text{k}\Omega \dfrac{1 + jf/(5\times10^4)}{1 + jf/10^6}$.



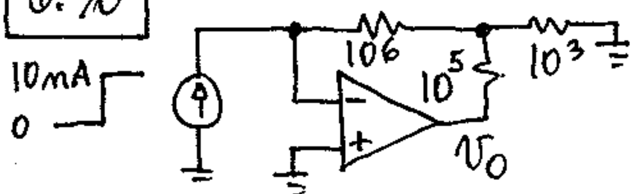
6.29

$f_b = 5$ Hz; $a_0 = 200$ V/mV; $f_t = 1$ MHz, $r_0 = 75\ \Omega$; $f_0 = 1/2\pi RC = 1$ kHz. $Z_o = \dfrac{r_o}{1+T}$.





6.30



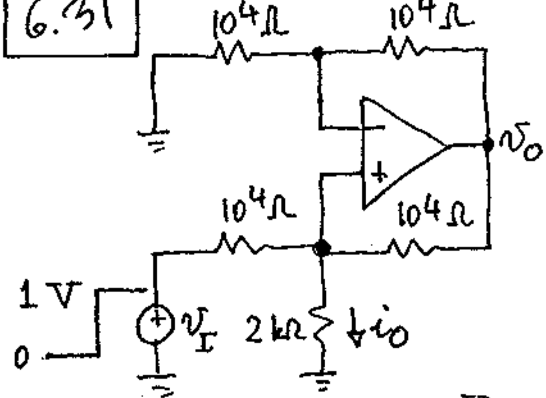
$V_{om} = -10^8 \times 10^{-8} = -1$ V

$\beta \cong 1/101$ V/V

$f_B \cong 10$ kHz.

$\tau = 1/2\pi f_B \cong 16\ \mu$s. $V_{om}(\text{crit}) \cong 80$ V $\Rightarrow$ small-signal limited. $v_O(t) = -1(1 - e^{-t/16\mu s})$ V.

6.31



$\beta = \frac{10}{10+10} - \frac{10\|2}{10\|2+10}$

$= 5/14$ V/V

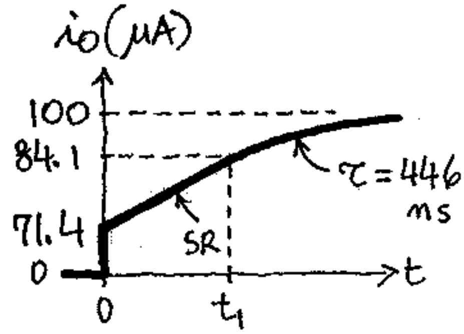
$f_A = \beta f_t = 357$ kHz

$\tau = 1/2\pi f_A = 446$ ns.

$V_{om} = (1+10/10)V_{pm} = 2R_L \frac{V_{im}}{R_1} = 2\times2\times1/10 = 0.4$ V;

$V_{om(crit)} = SR\times\tau = 0.5\times10^6\times446\times10^{-9} = 0.223\text{ V} \Rightarrow$ SR-limited for $v_O < 0.4 - 0.223 = 0.177$ V, small-signal limited thereafter. $i_O = v_P/R_L = (1/R_L)\frac{10\|2}{10\|2+10}(v_I + v_O)$

$\therefore i_O = (v_I + v_O)/(14\text{ k}\Omega)$.

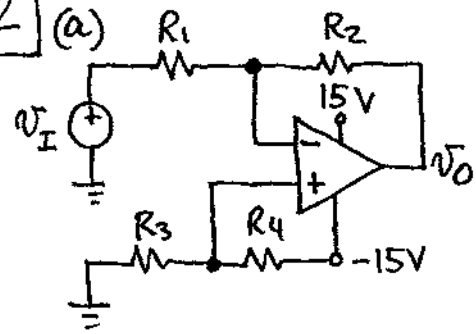


$i_O(0) = 1/14 = 71.4\ \mu$A

$i_O(t_1) = \frac{1+0.177}{14} = 84.1\ \mu$A

For $0 \le t \le t_1$, $v_O(t) = 0.5\times10^6 t$, so t_1 is found as $t_1 = 0.177/(0.5\times10^6) = 354$ ns.

6.32 (a)



Maximize β by avoiding any additional resistances at the inverting input.

Use $R_1 = R_2 = R_4 = 100$ kΩ, $R_3 = 20$ kΩ.

(b) $\beta = 0.5$ V/V, $f_A = 500$ kHz, FPB = 6.1 kHz.

6.33 $T = 1/(250\times10^3) = 4\mu s$. During $T/2$ (or $2\mu s$) the output changes by $|A|\times 2V_{im} = 2\times2\times2.5 = 10\text{V}$. Consequently, $SR = \Delta v_O/\Delta t = (10\text{ V})/(2\mu s) = 5\text{V}/\mu s$. By Eq. (6.27), $V_{om(crit)} = SR/2\pi f_A \Rightarrow f_A = SR/2\pi V_{om(crit)} \cong 2\text{ MHz}$. Small-signal bandwidth $= f_A = 2\text{MHz}$; large-signal bandwidth $= SR/[2\pi V_{om}] = 5\times10^6/[2\pi\times2\times3.5\times\sqrt{2}] \cong 80\text{kHz}$.
$\Rightarrow$ Useful bandwidth $= 80$ kHz, SR limited.

6.34 $f_1 = f_2 = \beta f_t = 10^6/\sqrt{10^3} = 31.6\text{ kHz}$; $\tau_1 = \tau_2 = 1/2\pi\times31.6\times10^3 \cong 5\mu s$. The output of OA_1 is $v_1 = (31.6\text{ mV})(1-e^{-t/(5\mu s)})$, without any SR limiting effects because $31.6\text{ mV} < 80\text{ mV}$.
The initial rate of change of v_O is $dv_O/dt|_{t=0} = A_{02}\times dv_1/dt|_{t=0} = 31.6\times\frac{31.6\text{ mV}}{5\mu s} = 0.2\text{ V/s}$. Since this is less than $0.5\text{ V}/\mu s$, there are no SR limiting effects at the output of OA_2 either. We can therefore apply linear analysis techniques (Laplace Xform).
$v_O(t) = \mathcal{L}^{-1}V_o(s)$, $V_o(s) = A(s)V_i(s) = \frac{10^3}{[1+s/(2\pi f_1)]^2}\times\frac{10^{-3}}{s} = \frac{1}{s[1+s/(2\times10^5)]^2} = \frac{4\times10^{10}}{s[s+2\times10^5]^2} = \frac{A_1}{s} + \frac{A_2}{[s+2\times10^5]^2} + \frac{A_3}{s+2\times10^5}$. We have

(6.16)

$$A_1 = V_o(s) \times s \Big|_{s=0} = 1$$

$$A_2 = V_o(s) \times [s + 2\times10^5]^2 \Big|_{s=-2\times10^5} = \frac{1}{s}\Big|_{s=-2\times10^5}$$

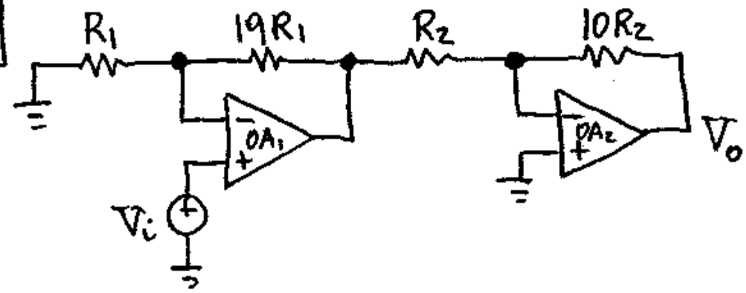
$$= -2\times10^5$$

$$A_3 = \frac{d}{ds}\frac{1}{s}\Big|_{s=-2\times10^5} = -1$$

$$v_o(t) = [A_1 + (A_2 t + A_3)e^{-t/5\mu s}]u(t)$$

$$= [1 - (2\times10^5 t + 1)e^{-t/5\mu s}]u(t) \text{ V.}$$

6.35



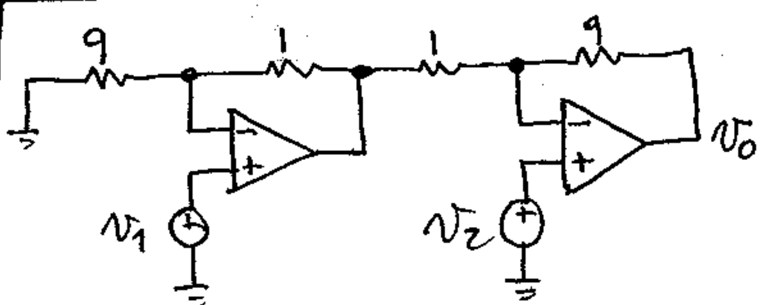
$$SR_2 \geq 2\pi f V_{o2m} = 2\pi\times10^5\times5\times\sqrt{2} = 4.44 \text{ V}/\mu s$$

$$SR_1 \geq SR_2/10 = 0.44 \text{ V}/\mu s$$

$$f_{t1} \geq f/\beta_1 = 10^5\times20 = 2 \text{ MHz}$$

$$f_{t2} \geq f/\beta_2 = 10^5\times11 = 1.1 \text{ MHz.}$$

6.36



$\beta_1 = 0.9$ V/V, $\beta_2 = 0.1$ V/V, $\omega_1 = 2\pi\beta_1 f_{t1} = 5.65\times10^6$ rad/s,

$\omega_2 = 2\pi\beta_2 f_{t2} = 0.628\times10^6$ rad/s.

$$V_o(s) = \frac{10}{1+s/\omega_2}\left[V_2(s) - \frac{1}{1+s/\omega_1}V_1(s)\right] = \frac{10\omega_2}{s+\omega_2}\left[V_2 - \frac{\omega_1}{s+\omega_1}V_1\right].$$

(6.17)

(a) Let $v_2(t) = V_{im}u(t)$, so that $V_2(s) = V_{im}/s$.

$$V_0(s) = \frac{10\omega_2}{s+\omega_2} \times \frac{V_{im}}{s} = \frac{A_0}{s} + \frac{A_2}{s+\omega_2}$$

$$A_0 = V_0(s) \times s \Big|_{s=0} = 10\, V_{im}$$

$$A_2 = V_0(s) \times (s+\omega_2) \Big|_{s=-\omega_2} = -10\, V_{im}$$

$$v_0(t) = \mathcal{L}^{-1} V_0(s) = \mathcal{L}^{-1}\left[\frac{10V_{im}}{s} - \frac{10V_{im}}{s+\omega_2}\right]$$

$$= 10V_{im}\left(1-e^{-\omega_2 t}\right)u(t) = 10V_{im}\left[1-e^{-t/(1.59\mu s)}\right]u(t).$$

(b) Let $V_1(s) = V_{im}/s$. Then,

$$V_0(s) = \frac{-10\omega_1\omega_2}{(s+\omega_2)(s+\omega_1)} \times \frac{V_{im}}{s} = -10V_{im}\left[\frac{1}{s} + \frac{1/8}{s+\omega_1} - \frac{9/8}{s+\omega_2}\right]$$

$$v_0(t) = -10V_{im}\left[1 + \frac{1}{8}e^{-\omega_1 t} - \frac{9}{8}e^{-\omega_2 t}\right]u(t)$$

$$= -10V_{im}\left[1 + \frac{1}{8}e^{-t/(177ms)} - \frac{9}{8}e^{-t/(1.59\mu s)}\right]u(t).$$

(c) $V_1(s) = V_2(s) = V_{im}/s$.

$$V_0(s) = \frac{10\omega_2}{s+\omega_2}\left[1 - \frac{\omega_1}{s+\omega_1}\right] \times \frac{V_{im}}{s} = 10V_{im}\frac{\omega_2}{(s+\omega_1)(s+\omega_2)}$$

$$= \frac{10}{8}V_{im}\left[\frac{1}{s+\omega_2} - \frac{1}{s+\omega_1}\right]$$

$$v_0(t) = 1.25V_{im}\left[e^{-t/(1.59\mu s)} - e^{-t/(177ms)}\right]u(t).$$

6.37 (a) Let $A_1 = A_2 = 10$. To simplify offset nulling, use inverting configuration. Let $R_1 = R_3 = 10\,k\Omega$, $R_2 = R_4 = 100\,k\Omega$. With $v_I = 0$,

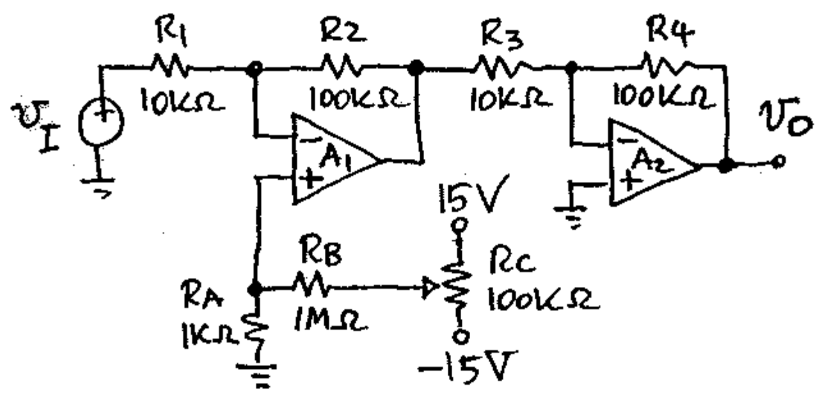
$$E_0 = \left(1+\frac{R_4}{R_3}\right)V_{OS2} - \frac{R_4}{R_3}\left(1+\frac{R_2}{R_1}\right)\left(V_{OS1} + V_{RA}\right) =$$

(6.18)

$11V_{OS2} - 110V_{OS1} - 110V_{RA}$. Thus,

$|V_{RA}| \le [110V_{OS1(max)} + 11V_{OS2(max)}]/110 = 11$ mV.

Impose $|V_{RA}|_{max} = 15$ mV to make sure. This can be achieved with $R_A = 1\,k\Omega$, $R_B = 1\,M\Omega$, $R_C = 100\,k\Omega$.



(b) $f_{A1} = f_{A2} = f_t/(1+100/10) = 4/11 = 365$ kHz. $f_A = 365(\sqrt{2}-1)^{1/2} = 235$ kHz. $FPB = 13\times10^6/(2\pi\times10) = 207$ kHz.

(c) $V_{om} = 100 \times \sqrt{2} \times 50\times10^{-3} = 7.07$ V. $f = 13\times10^6/(2\pi\times70.7) = 292$ kHz. Useful range is up to 235 kHz, small-signal limited.

$\boxed{6.38}$ $(R_2/R_1)/a_o = 10/200{,}000 = 5\times10^{-5}$; $f_a = f_t/a_o = 3\times10^6/200{,}000 = 15$ Hz; $f_A = f_t/(1+R_2/R_1) = 3\times10^6/(1+10) = 273$ kHz. Thus,

$$v_N(pk\text{-}pk) = (50\,\mu V)\sqrt{\frac{1+(f/15)^2}{1+[f/(273\times10^3)]^2}}.$$

This relation holds up to $f = SR/(2\pi V_{om}) = 13\times10^6/(2\pi\times5) = 414$ kHz, after which slew-rate limiting introduces distortion.

(6.19)

f	v_N (pk-pk)
1 Hz	50.1 μV
10 Hz	60.1 μV
100 Hz	337 mV
1 kHz	3.33 mV
10 kHz	33.3 mV
100 kHz	0.313 V
400 kHz	0.752 V

Above 414 kHz, v_N distorts somewhat, and its pk-pk amplitude approaches 0.91 V.

6.39 $\beta = R_1/(R_1+R_2) = 1/4$ V/V; $A_0 = 10^5 \times 4 = 0.4$ V/μA; $f_B = \beta f_t = (1/4)4 = 1$ MHz. $V_{om(crit)} = SR/2\pi f_B = 15\times10^6/(2\pi 10^6) = 2.39$ V; $V_{om} = 0.4\times10^6 \times 20\times10^{-6} = 8$ V $> V_{om(crit)} \Rightarrow$ slew-rate limited, and $f \le SR/2\pi V_{om} \cong 300$ kHz.

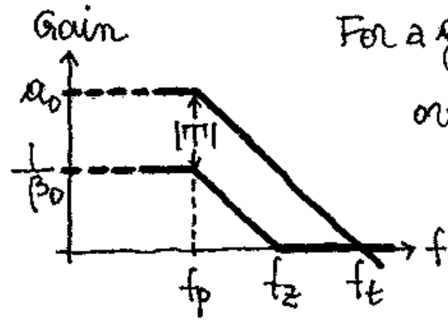
6.40 [illegible]

(b) $A_0 = 2$ V/V; $\beta = 0.5$; $V_{om(crit)}$ [illegible] mV; $V_{im(crit)} \cong 80$ mV

(d) $A_0 = -2$ V/V, $\beta = 1/3$; $V_{om(crit)}$ [illegible] 239 mV; $V_{im(crit)} \cong 120$ mV.

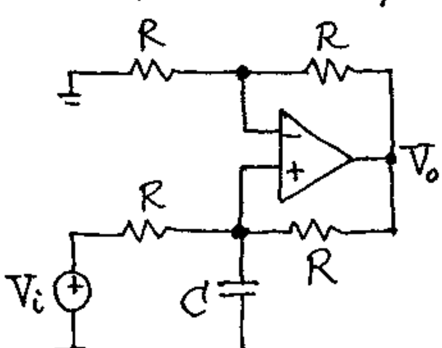
6.41 For this circuit, $A_0 = -8$ V/V, and $\beta = 1/13$ V/V. We want $f_B = \beta f_t \geq 1$ MHz, or $f_t \geq 13$ MHz, and $SR \geq 2\pi f V_{om} = 2\pi \times 10^6 \times 8 \times 1 \cong 50$ V/μs.

6.42 In the upper audio range we have $\frac{1}{\beta} \cong \left(1+\frac{R_2}{R_1}\right)\frac{1+jf/f_z}{1+jf/f_p}$, $1+\frac{R_2}{R_1} = 11$ V/V, $f_z = \frac{1}{2\pi(R_1 \| R_2)C_2} = 224$ kHz, $f_p = \frac{1}{2\pi R_2 C_2} = 20$ kHz.



For a gain error of less than 1% over the entire audio range we need $|T(j\,20\,\text{kHz})| \geq 100$, or $a_0 \geq 1{,}100$ V/V, and $f_t \geq a_0 f_p = 22$ MHz. Moreover, $SR \geq 2\pi \times (20\ \text{kHz}) \times (10\ \text{V}) = 1.3$ V/μs.

6.43 (a) $R = 2/(2\pi \times 10^3 \times 10 \times 10^{-9}) = 13.6\ \text{k}\Omega, 1\%$.



(b) To find β, suppress V_i, break the wire at the op amp output, and apply a test voltage V_t. Then, $\beta = (V_n - V_p)/V_t$:

$$\beta = \frac{R}{R+R} - \frac{R\|(1/sC)}{R+R\|(1/sC)} = \frac{1}{2} - \frac{1}{1+R/[R\|(1/s)]}$$

$$= \frac{1}{2} - \frac{1}{2+sRC} = \frac{1}{2}\,\frac{jf/f_0}{1+jf/f_0}, \quad f_0 = 1\ \text{kHz}.$$

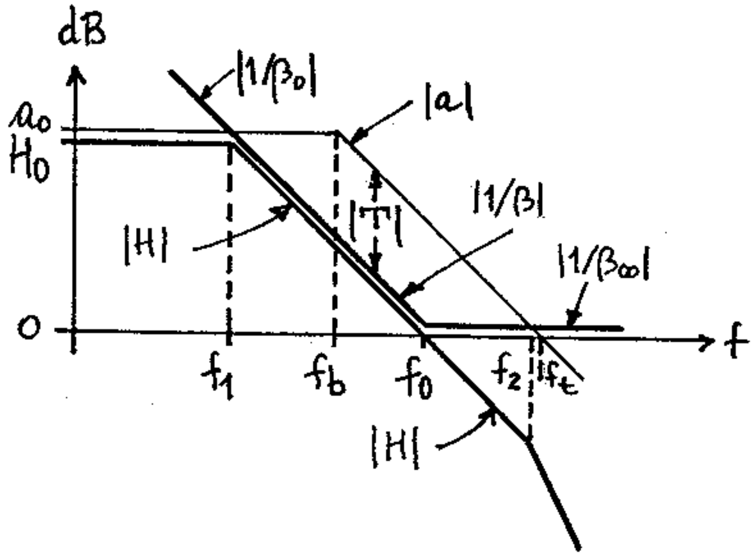
(6.21)

$\frac{1}{\beta} = 2\frac{1+jf/f_0}{jf/f_0}$; $\frac{1}{\beta_0} = \frac{1}{jf/2f_0}$; $\frac{1}{\beta_\infty} = 2$ V/V.

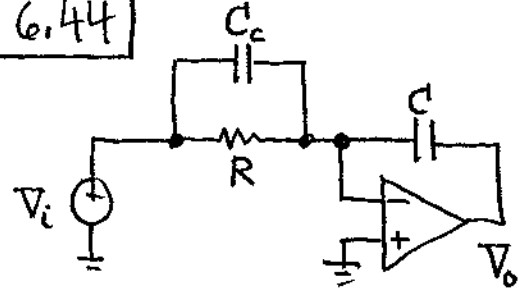
The $|1/\beta_0|$ curve intercepts the $|a|$ curve at a frequency f_1 such that $1/(f_1/2f_0) = a_0$, or $f_1 = 2f_0/a_0 = 2\times10^3/200{,}000 = 0.01$ Hz ; the $|1/\beta_\infty|$ curve intercepts the $|a|$ curve at a frequency f_2 such that $2 = 1/(f_2/f_t)$, or $f_2 = f_t/2 = 500$ kHz.

The transfer function is $H(jf) = H_{ideal} \times 1/(1+1/T) = [1/(jf/f_0)]/(1+1/T)$. For $f_1 \ll f \ll f_2$, where $|T| \gg 1$, we have $H \cong H_{ideal}$. For $f \ll f_1$, $H \to [1/(jf/f_0)]a_0\beta_0 = a_0/2$. For $f \gg f_2$, $H \to [1/(jf/f_0)]\,a_\infty\beta_\infty = \frac{1}{jf/f_0} \times \frac{1}{jf/f_2}$, indicating a breakpoint at f_1 and another at f_2:

$$H(jf) = \frac{a_0/2}{[1+jf/f_1][1+jf/f_2]} = \frac{10^5}{[1+jf/0.01][1+jf/(5\times10^5)]}$$



6.44



Let $Z_1 = R \| (1/sC_c)$

$$= \frac{R}{1+sRC_c},$$

$Z_2 = 1/sC$, $a \cong \omega_t/s$.

$$H = \frac{V_o}{V_i} = -\frac{Z_2}{Z_1}\,\frac{1}{1+\left(1+\frac{Z_2}{Z_1}\right)\frac{1}{a}} = -\frac{1+sRC_c}{sRC}\,\frac{1}{1+\left(1+\frac{1+sRC_c}{sRC}\right)\frac{s}{\omega_t}}$$

$$= -\frac{1}{s/\omega_0} \times \frac{1+sRC_c}{1+\frac{s}{\omega_t}+\frac{\omega_0}{\omega_t}+s\frac{\omega_0 RC_c}{\omega_t}} \cong -\frac{1}{s/\omega_0} \times \frac{1+sRC_c}{1+s\frac{1+\omega_0 RC_c}{\omega_t}}.$$

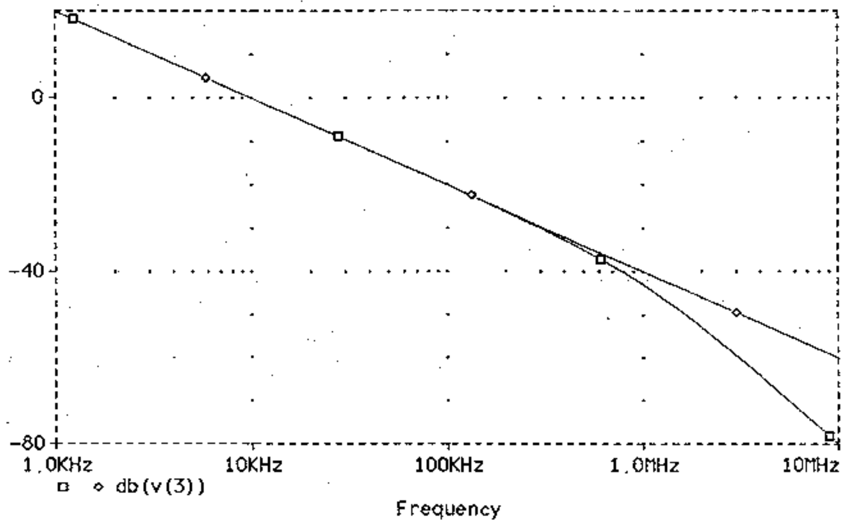
(b) To make the error function unity, impose $RC_c = (1+\omega_0 RC_c)/\omega_t$. Using $R = 1/\omega_0 C$, this gives $C_c = C/(f_t/f_0 - 1)$.

(c) $C = 1\text{nF}$, $R = 1/2\pi f_0 C = 15.91\ \text{k}\Omega$, $C_c = C/(10^6/10^4 - 1) = 10.1\ \text{pF}$.

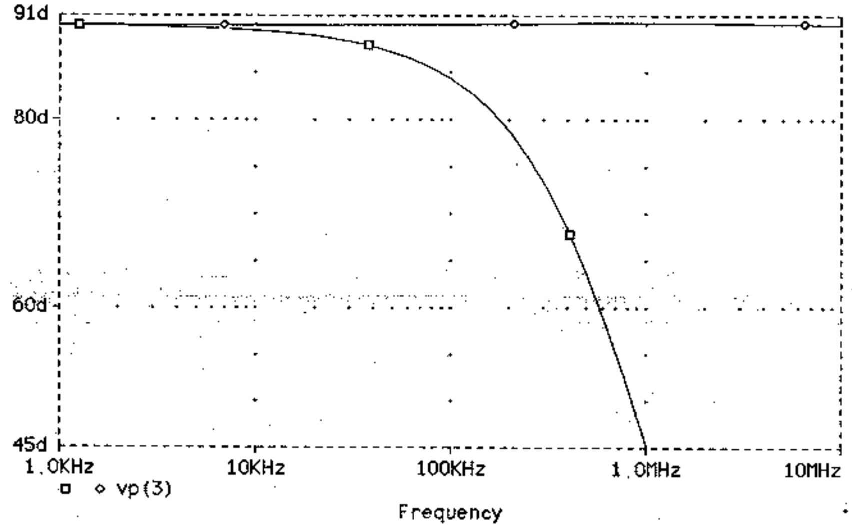
The following PSpice files show the response first without and then with C_c.

```
Problem 6.44
Vi 1 0 ac 1V
R 1 2 15.91k
Cc 1 2 1fF
C 2 3 1nF
eopamp 3 0 Laplace {V(0,2)}={200k/(1+s/31.42)}
.ac dec 10 1k 10Meg
.probe
.end
Problem 6.44
Vi 1 0 ac 1V
R 1 2 15.91k
Cc 1 2 10.1pF
C 2 3 1nF
eopamp 3 0 Laplace {V(0,2)}={200k/(1+s/31.42)}
.ac dec 10 1k 10Meg
.probe
.end
```

(6.23)

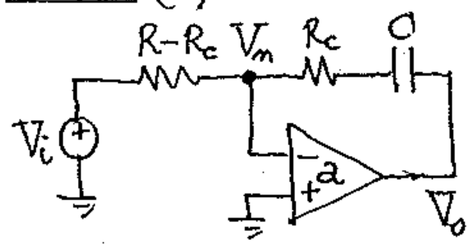


0
-40
-80
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
db(v(3))
Frequency



91d
80d
60d
45d
1.0KHz
10KHz
100KHz
1.0MHz
10MHz
vp(3)
Frequency

6.45 (a)



$a \cong \frac{\omega_t}{s}$; superposition:

$$V_n = \frac{(R_c + 1/sC)V_i + (R-R_c)V_o}{R + 1/sC}$$

$$= \frac{(sR_cC+1)V_i + s(R-R_c)CV_o}{1+sRC}$$

$$V_o = -aV_n = -\frac{\omega_t}{s}\,\frac{(sR_cC+1)V_i + s(R-R_c)CV_o}{1+sRC}$$; collecting,

$$H = \frac{V_o}{V_i} = -\frac{sR_cC+1}{sRC(s/\omega_t+1) + s(1/\omega_t - R_cC)}$$

(b) Imposing $1/\omega_t = R_cC$, or $R_c = 1/2\pi f_t C$ has the double effect of eliminating the second s-term in the denominator, and simplifying H to $H = -(s/\omega_t+1)/[sRC(s/\omega_t+1)] = -1/sRC$.

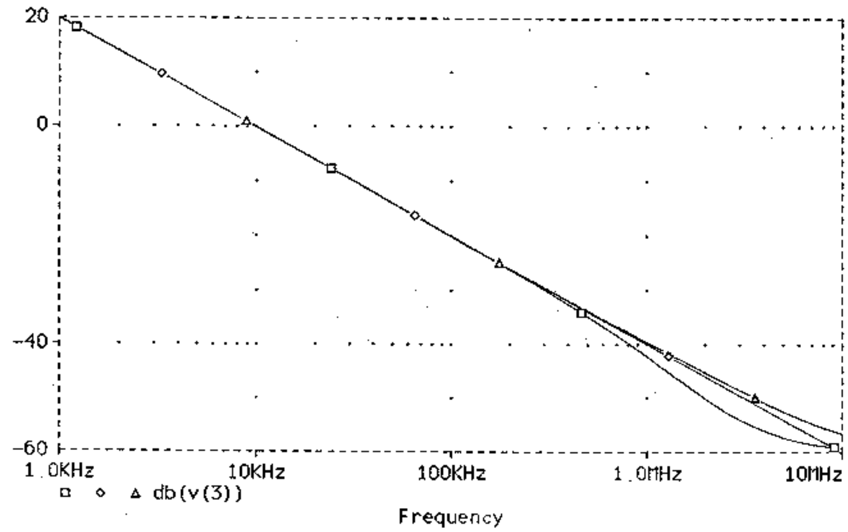
(c) To contain the effect of $r_o \neq 0$, make $R_c \gg r_o$, e.g. $R_c = 1\ \text{k}\Omega$. Then, $R = (f_t/f_o)R_c = (10^6/10^4)10^3 = 100\ \text{k}\Omega$, so $R - R_c = 99\ \text{k}\Omega$, and $C = 1/2\pi f_o R = 159.15\ \text{pF}$.

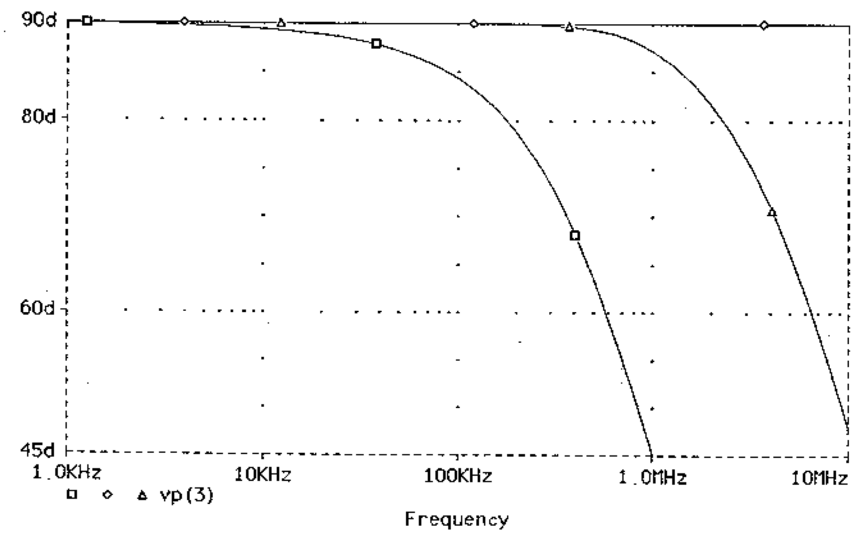
The following PSpice code shows the uncompensated response, as well as the compensated response for the ideal case $r_o = 0$ and the more practical case $r_o = 100\ \Omega$.

6.25

```
Problem 6.45
Vi 1 0 ac 1V
R 1 2 100k
Rc 2 4 1m
C 4 3 159.154pF
eopamp 5 0 Laplace {V(0,2)}={200k/(1+s/31.42)}
ro 5 3 100
.ac dec 10 1k 10Meg
.probe
.end
Problem 6.45
Vi 1 0 ac 1V
R 1 2 99k
Rc 2 4 1k
C 4 3 159.154pF
eopamp 5 0 Laplace {V(0,2)}={200k/(1+s/31.42)}
ro 5 3 1m
.ac dec 10 1k 10Meg
.probe
.end
Problem 6.45
Vi 1 0 ac 1V
R 1 2 99k
Rc 2 4 1k
C 4 3 159.154pF
eopamp 5 0 Laplace {V(0,2)}={200k/(1+s/31.42)}
ro 5 3 100
.ac dec 10 1k 10Meg
.probe
.end
```

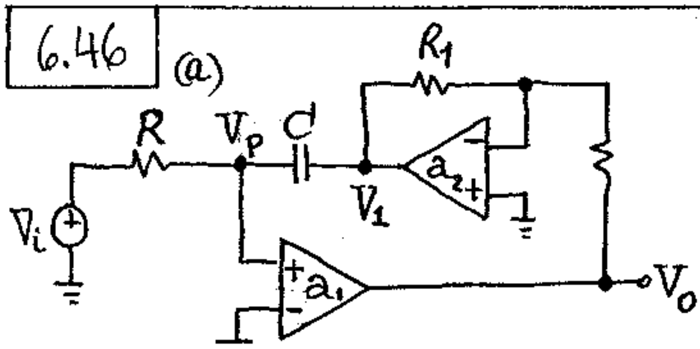
6.26





6.46

(a)



$V_o = a_1 V_P$, $V_P = \frac{1}{1+s/\omega_0} V_i + \frac{s/\omega_0}{1+s/\omega_0} V_1$, $V_1 = -\frac{1}{1+s/(\omega_{t_2}/2)} V_o$, $a_1 \cong \omega_{t_1}/s$. Combining and letting $\omega_{t_1} = \omega_{t_2} = \omega_t$ gives

(6.27)

$$H = \frac{V_o}{V_i} = +\frac{1}{s/\omega_0} \times \frac{1 + s/(\omega_t/2)}{1 + \frac{\omega_0}{\omega_t} + \frac{s}{\omega_t}\left(1 + \frac{\omega_0}{\omega_t/2}\right) + \frac{s^2}{\omega_t^2/2}}$$

$$\cong +\frac{1}{s/\omega_0} \times \frac{1 + s/(\omega_t/2)}{1 + \frac{s}{\omega_t} + \frac{s^2}{\omega_t^2/2}}.$$

Letting $s/\omega_t = jf/f_t = jx$, the error function is

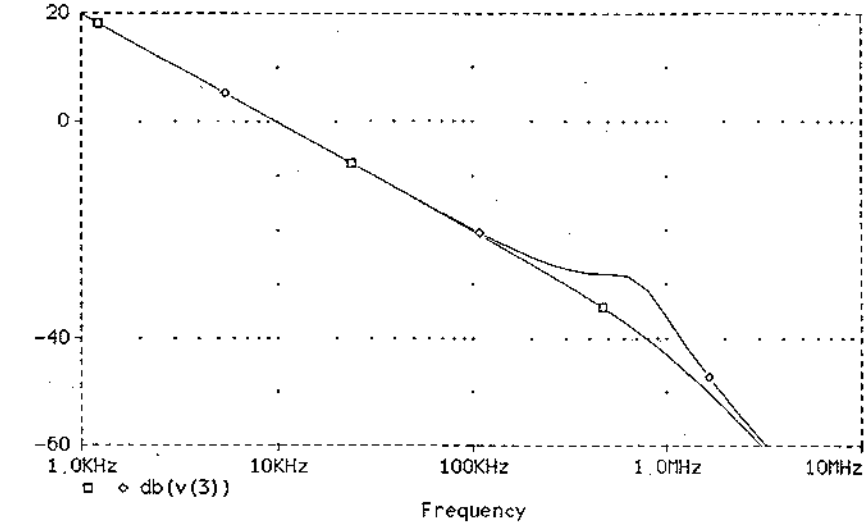
$$\frac{1 + j2x}{1 - 2x^2 + jx} = \frac{(1 + j2x)(1 - 2x^2 - jx)}{(1 - 2x^2)^2 + x^2} = \frac{1 + jx - j2x^3}{1 - 3x^2 + 4x^4}.$$

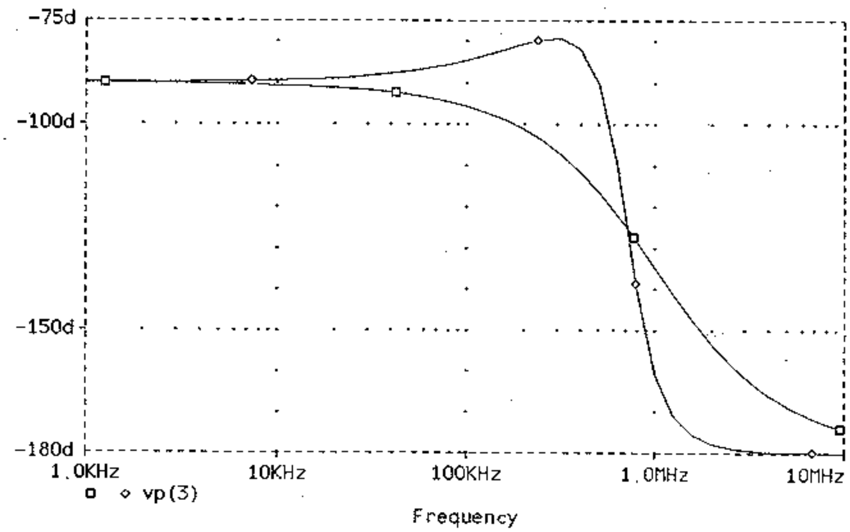
For $x \ll 1$ this reduces approximately to $1 + jx$, indicating $\epsilon_\phi \cong x = f/f_t$.

(b) The following PSpice code shows the response without compensation (OA_2 ~ ideal) and with compensation.

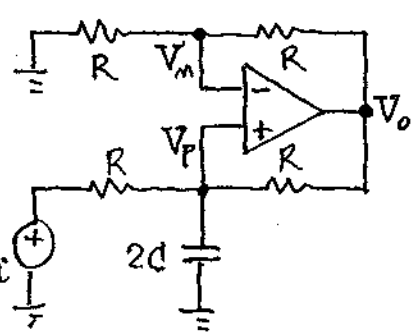
```
Problem 6.46
Vi 1 0 ac 1V
R 1 2 15.9154k
C 2 5 1nF
eoa1 3 0 Laplace {V(2,0)}={200k/(1+s/31.42)}
r1 3 4 10k
r2 4 5 10k
eoa2 5 0 Laplace {V(0,4)}={1G}
.ac dec 10 1k 10Meg
.probe
.end
Problem 6.46
Vi 1 0 ac 1V
R 1 2 15.9154k
C 2 5 1nF
eoa1 3 0 Laplace {V(2,0)}={200k/(1+s/31.42)}
r1 3 4 10k
r2 4 5 10k
eoa2 5 0 Laplace {V(0,4)}={200k/(1+s/31.42)}
.ac dec 10 1k 10Meg
.probe
.end
```

6.28





$\boxed{6.47}$ (a)



$V_o = a(V_p - V_n)$; $V_n = V_o/2$;

$$V_p = \frac{R \| (1/2sC)}{R + R\|(1/2sC)}(V_i + V_o)$$

$$\frac{V_p}{V_i + V_o} = \frac{1/2}{1+sRC} = \frac{1/2}{1+s/\omega_0}, \quad \omega_0 = \frac{1}{RC}$$

Letting $a = \omega_t/s$ & combining,

$$H = \frac{V_o}{V_i} = \frac{1}{s/\omega_0}\,\frac{1}{1+2\omega_0/\omega_t + 2s/\omega_t} \cong \frac{1}{jf/f_0} \times \frac{1}{1+jf/(f_t/2)}$$

$\varepsilon_\phi \cong -f/(f_t/2)$ for $f \ll f_t/2$

(6.29)

We now have

$$V_p = \frac{R\|(R_c+1/s2C)}{R+R\|(R_c+1/s2C)}(V_i+V_o) = \frac{1}{2}\,\frac{1+s2R_cC}{1+s(R+2R_c)C}(V_i+V_o)$$

Substituting and collecting,

$$H = \frac{V_o}{V_i} = \frac{1}{s/\omega_0}\,\frac{1+2sR_cC}{1+2\frac{\omega_0}{\omega_t}+2\frac{\omega_0}{\omega_t}s(R+2R_c)C}$$

$$\cong \frac{1}{s/\omega_0}\,\frac{1+2sR_cC}{1+2s\frac{\omega_0}{\omega_t}(R+2R_c)C}.$$

To drive the error term to unity, impose

$R_c = \frac{\omega_0}{\omega_t}(R+2R_c)$, or $R_c = R/(\omega_t/\omega_0-2)$.

6.48 The closed-loop gain of the second op amp is $A_2 = [(1+R_2/R_1)-R_2/R_1]/(1+s/\omega_2)] = 1/(1+s/\omega_2)$, $\omega_2 = \beta_2\omega_{t_2} = [R_1/(R_1+R_2)]\omega_{t_2}$. The first op amp gives

$$V_o = -a_1\left[\frac{1}{1+s/\omega_0}V_i + \frac{s/\omega_0}{1+s/\omega_0}A_2V_o\right],$$

$\omega_0 = 1/RC$. Substituting A_2 and $a_1 = \omega_1/s$, $\omega_1 = \omega_{t_1}$, we get, after collecting,

$$H = \frac{V_o}{V_i} = \frac{-1}{s/\omega_0}\,\frac{1+s/\omega_2}{1+\frac{\omega_0}{\omega_1}+\frac{s}{\omega_1}\left(1+\frac{\omega_0}{\omega_2}\right)+\frac{s^2}{\omega_1\omega_2}}.$$ For $\omega_0 \ll \omega_1$

and $\omega_0 \ll \omega_2$, this simplifies to

$$H = \frac{-1}{s/\omega_0}\,\frac{1+s/\omega_2}{1+s/\omega_1+s^2/\omega_1\omega_2} = \frac{-1}{jf/f_0}\,\frac{1+jf/(\beta_2 f_{t2})}{1-f^2/(\beta_2 f_{t_1}f_{t_2}+jf/f_{t_1}}.$$

For $f_{t_1}=f_{t_2}=f_t$ and $R_1=R_2$ ($\Rightarrow \beta_2=0.5$) we get

(6.30)

Error function $= \dfrac{1+2jf/f_t}{1-2(f/f_t)^2+jf/f_t} = \dfrac{1+j2x}{1-2x^2+jx}$

$= \dfrac{(1+j2x)(1-2x^2-jx)}{(1-2x^2)^2+x^2} = \dfrac{1+jx-j4x^3}{1-3x^2+4x^4}$, indicating that for $x \ll 1$ $(f \ll f_t)$ we have $\epsilon_\phi \cong +(f/f_t)$. Compared to the ordinary inverting integrator, which gives a negative ϵ_ϕ (see Eq. 6.35), the present integrator gives a positive ϵ_ϕ (of the type of Eq. 6.40); it can be used to compensate for the negative ϵ_ϕ's of other integrators or amplifiers in the same loop.

$\boxed{6.49}$ The second op amp provides a closed-loop gain of $A_2 = \left[\left(1+\dfrac{R_2}{R_1}\right) - \dfrac{R_2}{R_1}\right]\dfrac{1}{1+s/\omega_2} = \dfrac{1}{1+s/\omega_2}$, $\omega_2 =$

$\beta_2\omega_{t2} = \dfrac{R_1}{R_1+R_2}\omega_{t2}$. The first op amp gives $V_o = a_1(V_{P1}-V_{N1})$, where $a_1 \cong \omega_1/s$, $\omega_1 = \omega_{t1}$, $V_{N1} = (1/2)A_2V_o$, and $V_{P1} = [0.5/(1+s/\omega_0)](V_i+V_o)$, $\omega_0 = \dfrac{1}{RC}$. Combining and collecting gives

$$H = \frac{V_o}{V_i} = \frac{1}{s/\omega_0}\,\frac{1+\frac{s}{\omega_2}}{2\frac{\omega_0}{\omega_1} + 2\frac{s}{\omega_1}\left(1+\frac{\omega_0}{\omega_2}\right) - \frac{\omega_0}{\omega_2} + 2\frac{s^2}{\omega_1\omega_2} + 1}$$

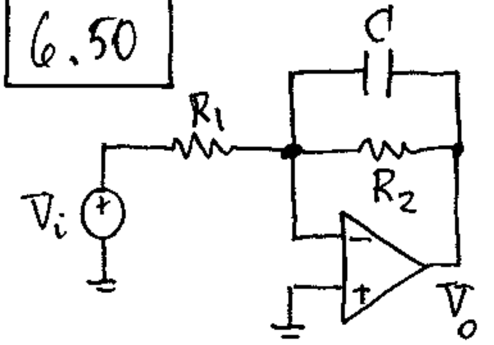
$$\cong \frac{1}{s/\omega_0}\,\frac{1+s/\omega_2}{1+2\frac{s}{\omega_1}+2\frac{s^2}{\omega_1\omega_2}}.$$

For $\omega_{t1} = \omega_{t2} = \omega_t$ and $R_1 = R_2$ $(\Rightarrow \beta = 0.5\ V/V)$

(6.31) the error function becomes

$$\frac{1+jf/0.5f_t}{1-(f/0.5f_t)^2+jf/0.5f_t}$$; this is of the same type of Eq. (6.38), but with $f_t/2$ instead of f_t. We thus conclude that $\varepsilon_\phi \cong -(f/0.5f_t)^3$ for $f \ll f_t/2$.

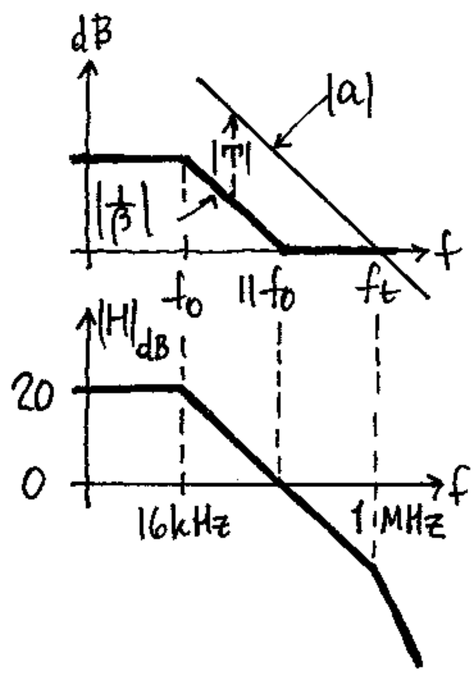
6.50



let $Z_2 = R_2 // (1/sC) = \frac{R_2}{1+jf/f_0}$, $f_0 = \frac{1}{2\pi R_2 C_2} =$ 16 kHz. Then

$$\frac{1}{\beta} = 1 + \frac{Z_2}{R_1} = 1 + \frac{10}{1+jf/f_0}$$

$$= 11\,\frac{1+jf/(11f_0)}{1+jf/f_0}$$



H has a pole at f_0 and another at f_t:

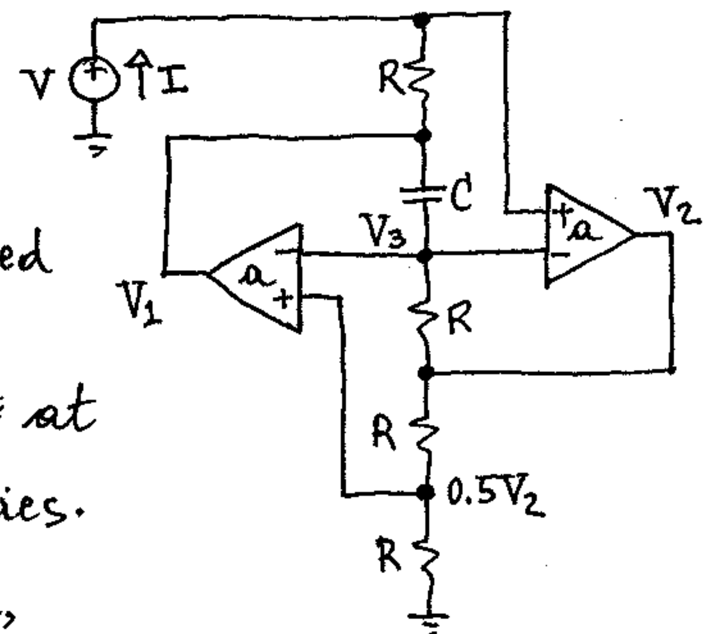
$$H = \frac{-10\ \text{V/V}}{\left(1+j\frac{f}{16\,\text{kHz}}\right)\left(1+j\frac{f}{1\,\text{MHz}}\right)}$$

6.51 $\beta = 0.5$ V/V; $\beta f_t = 0.5 f_t$; $H = H_{ideal}\frac{1}{1+1/T}$

$H = \frac{1-jf/f_0}{1+jf/f_0} \times \frac{1}{1+jf/0.5f_t}$. $|H|$ is constant only up to $0.5f_t$, after which it rolls off with f.

We also have a phase error $\varepsilon_\phi = -\tan^{-1}(f/0.5f_t)$. For instance, if $f_0 = 10$ kHz and $f_t = 1$ MHz, we get $H(j10^4\,\text{Hz}) = 0.9998\ \text{V/V}\ \angle -91.15^\circ$ instead the ideal value $1\ \text{V/V}\ \angle -90^\circ$.

6.52



For simplicity assume equal R's and matched OAs, so that $a_1 = a_2 = a = \frac{\omega_t}{s}$ at high frequencies.

By inspection,

$V_1 = a(0.5V_2 - V_3)$, $V_2 = a(V - V_3)$. By the superposition principle,

$$V_3 = \frac{V_2 + (s/\omega_0)V_1}{1 + s/\omega_0}, \quad \omega_0 = \frac{1}{RC}.$$

System of 3 equations in 3 unknowns. Cramer's rule:

$$V_1 = \frac{0.5a^2(s/\omega_0 - 1)}{1 + a + (1 + a + 0.5a^2)s/\omega_0} V$$

$$I = \frac{V - V_1}{R} = \frac{1}{R}\frac{(1+a)s/\omega_0 + 1 + a + 0.5a^2}{1 + a + (1 + a + 0.5a^2)s/\omega_0} V$$

Setting $a = \omega_t/s$ and exploiting the fact that $\omega_0 \ll \omega_t$, we obtain

$$Z = \frac{V}{I} \cong Ls\frac{0.5 + s/\omega_t + (s/\omega_t)^2}{0.5 + s/\omega_t + s^2/\omega_0\omega_t + s^3/\omega_0\omega_t^2}, \quad L = R^2C.$$

(6.33)

As a check: $Z(s\to 0) = Ls$, and $Z(\omega_t \to \infty) = Ls$.
Moreover, $Z(s\to\infty) = R$, as expected using physical insight. The error function departs significantly from 1 as f is increased, and it exhibits even peaking, as revealed by Fig. 1.

```
Problem 6.52: L simulator using ideal op amps
Ii 0 1 ac 1
R1 1 2 7.96k
C2 2 3 10n
R3 3 4 7.96k
R4 4 5 7.96k
R5 5 0 7.96k
e1 2 0 5 3 1G
e2 4 0 1 3 1G
.ac dec 100 1k 10Meg
.probe
.end
Problem 6.52: L simulator using OAs with ft=1 MHz
Ii 0 1 ac 1
R1 1 2 7.96k
C2 2 3 10n
R3 3 4 7.96k
R4 4 5 7.96k
R5 5 0 7.96k
e1 2 0 Laplace {V(5,3)}={1Meg/(1+s/6.283)}
e2 4 0 Laplace {V(1,3)}={1Meg/(1+s/6.283)}
.ac dec 100 1k 10Meg
.probe
.end
```

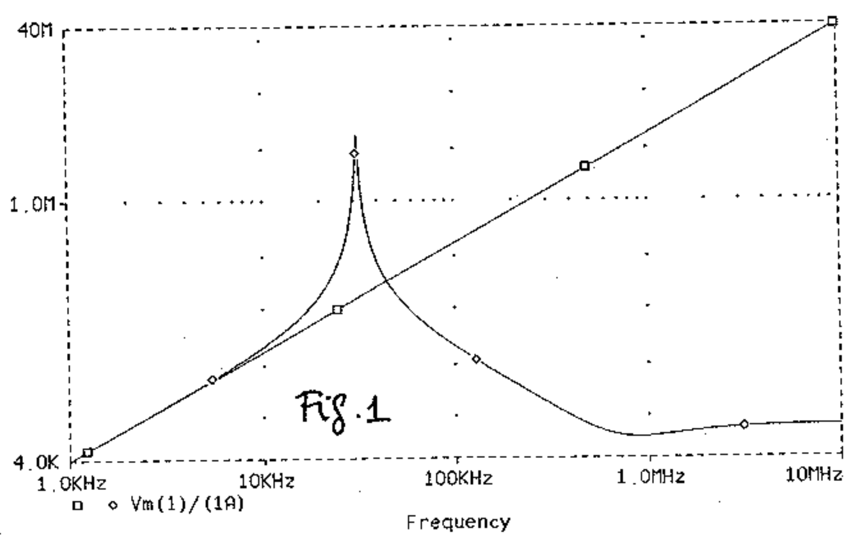


Fig. 1

(6.34)

It is intriguing that in spite of the poor inductance behavior at high frequencies, the response of the actual DABP filter is fairly close to the ideal over a far wider range (Fig. 2.) As shown in greater detail in Fig. 3, the effect is a downshift from $f_0 = 2.0$ kHz to $f_0 = 1.984$ kHz, which is readily compensated for using predistortion.

```
Problem 6.52: DABP using ideal op amps
Vi 10 0 ac 1
R 10 1 199k
C 1 0 10nF
R1 1 2 7.96k
C2 2 3 10n
R3 3 4 7.96k
R4 4 5 7.96k
R5 5 0 7.96k
e1 2 0 5 3 1G
e2 4 0 1 3 1G
.ac dec 100 0.1k 10Meg
.probe
.end
Problem 6.52: DABP using op amps with ft=1 MHz
Vi 10 0 ac 1
R 10 1 199k
C 1 0 10nF
R1 1 2 7.96k
C2 2 3 10n
R3 3 4 7.96k
R4 4 5 7.96k
R5 5 0 7.96k
e1 2 0 Laplace {V(5,3)}={1Meg/(1+s/6.283)}
e2 4 0 Laplace {V(1,3)}={1Meg/(1+s/6.283)}
.ac dec 100 0.1k 10Meg
.probe
.end
```

(6.35)

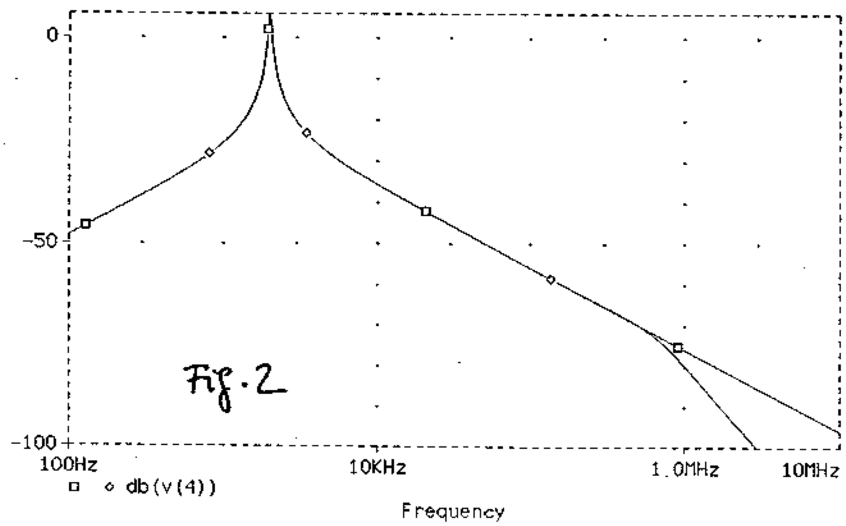


Fig. 2

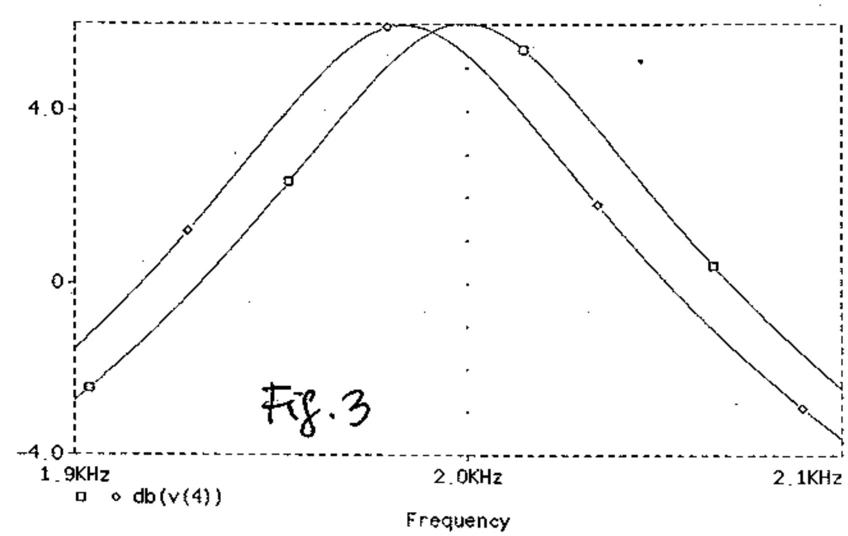


Fig. 3

(6.36)

$\boxed{6.53}$ For the LP filter of Fig. 3.23 we have:

$$H = \frac{A}{R_1C_1R_2C_2s^2 + [(1-A)R_1C_1 + R_1C_2 + R_2C_2]s + 1}.$$

Letting $\omega_0 = \dfrac{1}{\sqrt{R_1C_1R_2C_2}}$

$$\frac{1}{\omega_0 Q} = (1-K)R_1C_1 + R_1C_2 + R_2C_2$$

$$A = \frac{K}{1 + s/(\omega_t/K)} = \frac{K}{1 + Ks/\omega_t}$$

$$K = 1 + R_B/R_A$$

we get $1 - A = [(1-K) + Ks/\omega_t]/[1 + Ks/\omega_t]$.

Substituting,

$$H = \frac{\dfrac{K}{1+Ks/\omega_t}}{\left(\dfrac{s}{\omega_0}\right)^2 + \left[\dfrac{1-K+Ks/\omega_t}{1+Ks/\omega_t}R_1C_1 + R_1C_2 + R_2C_2\right]s + 1}$$

$$= \frac{K}{\left(\dfrac{s}{\omega_0}\right)^2 + \dfrac{s}{\omega_0 Q} + 1 + \dfrac{Ks}{\omega_t}\left[\left(\dfrac{s}{\omega_0}\right)^2 + (R_1C_1 + R_1C_2 + R_2C_2)s + 1\right]}$$

But, $R_1C_1 + R_1C_2 + R_2C_2 = \dfrac{1}{\omega_0 Q} + KR_1C_1 = \dfrac{1}{\omega_0 Q}\left(1 + QK\sqrt{\dfrac{R_1C_1}{R_2C_2}}\right)$

$$\therefore H(s) = \frac{K}{\left(\dfrac{s}{\omega_0}\right)^2 + \dfrac{1}{Q}\left(\dfrac{s}{\omega_0}\right) + 1 + \dfrac{Ks}{\omega_t}\left[\left(\dfrac{s}{\omega_0}\right)^2 + \dfrac{1}{Q}\left(\dfrac{s}{\omega_0}\right)\left(1 + QK\sqrt{\dfrac{R_1C_1}{R_2C_2}}\right) + 1\right]}.$$

(3.37)

$\boxed{6.54}$ Using the PSpice code shown, we find that the effects of finite GBPs are a steeper rolloff at high frequencies (Fig. 1) as well as a shift in f_0 from 1 kHz to 995 Hz, and a reduction in Q from 100 to 67 (Fig. 2). Changing the capacitances from 10 nF to 9.95 nF, and adding a compensating capacitance C_c = 50 pF in parallel with R_6 restores the desired response (Fig. 3).

```
Problem 6.54: SV filter with ideal OAs
Vi 1 0 ac 1
r1 1 3 1k
r2 3 6 299k
r3 1 2 15.92k
r4 2 8 15.92k
r5 2 4 15.92k
r6 4 5 15.92k
r7 6 7 15.92k
c1 5 6 10n
c2 7 8 10n
eoa1 4 0 3 2 1G
eoa2 6 0 0 5 1G
eoa3 8 0 0 7 1G
.ac dec 100 100 1Meg
.probe
.end
Problem 6.54: SV filter with 1-MHz OAs
Vi 1 0 ac 1
r1 1 3 1k
r2 3 6 299k
r3 1 2 15.92k
r4 2 8 15.92k
r5 2 4 15.92k
r6 4 5 15.92k
r7 6 7 15.92k
c1 5 6 10n
c2 7 8 10n
eoa1 4 0 Laplace {V(3,2)}={200k/(1+s/6.283)}
eoa2 6 0 Laplace {V(0,5)}={200k/(1+s/6.283)}
eoa3 8 0 Laplace {V(0,7)}={200k/(1+s/6.283)}
.ac dec 100 100 1Meg
.probe
.end
```

(6.38)

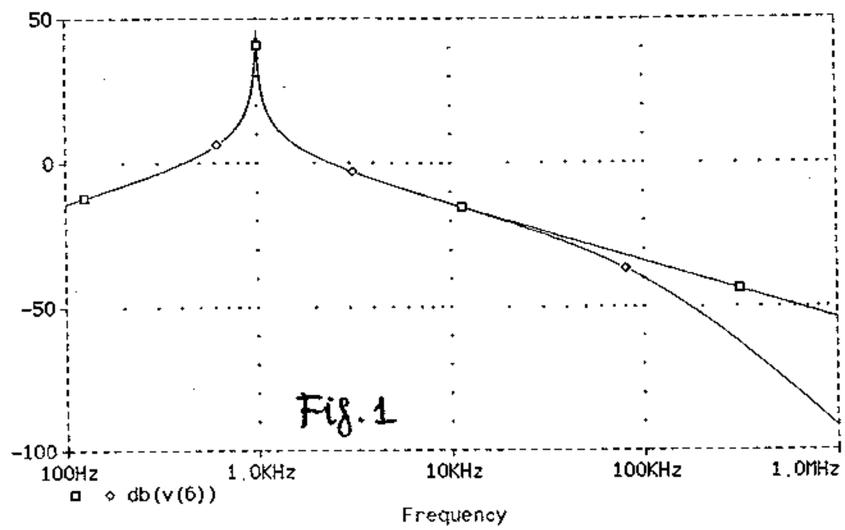


Fig. 1

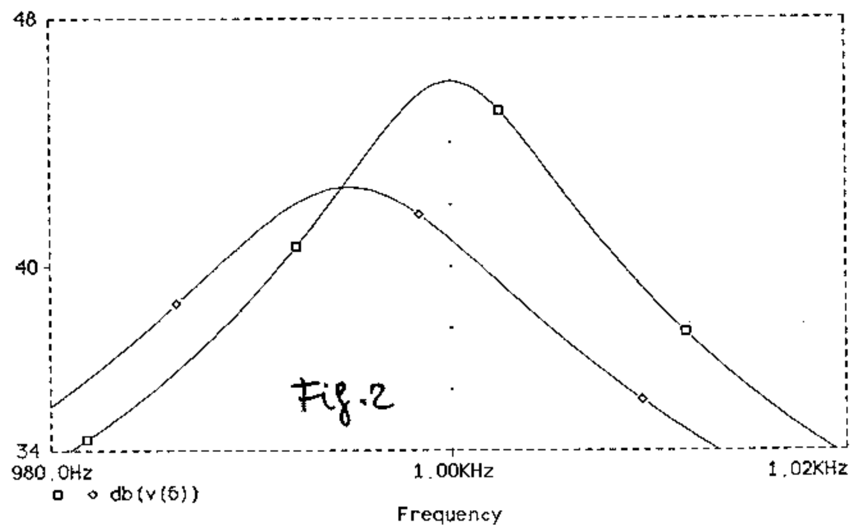


Fig. 2

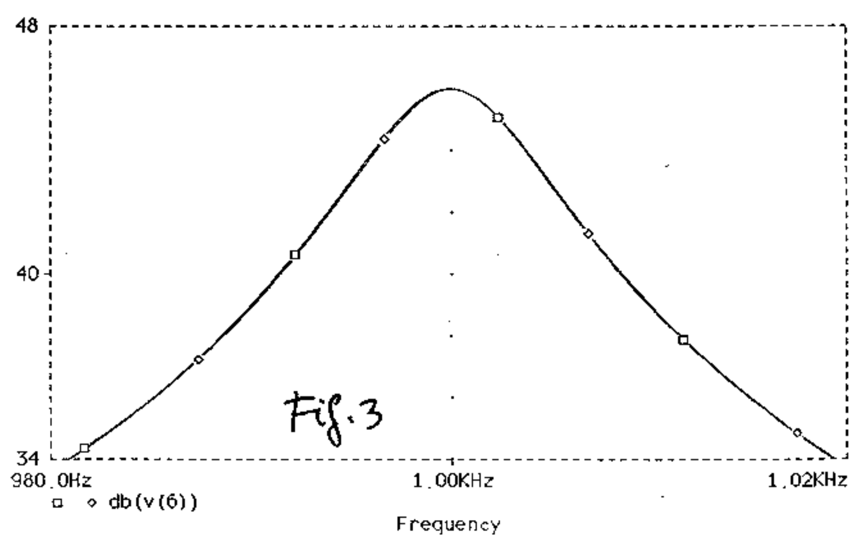


Fig. 3

$\boxed{6.55}$ Using the PSpice code shown, we find that the finite GBP has little effect on the location of the notch (Fig 1: $f_0 = 60$ Hz). The GBP of 1 MHz causes the actual response to roll off with frequency past 100 kHz (Fig 2).

```
Problem 6.55: Notch with ideal OA
Vi 0 1 ac 1
r1 1 2 26.526k
r2 2 4 26.526k
r3 3 0 13.263k
c1 1 3 100n
c2 3 4 100n
c3 2 5 200n
ra 6 0 10k
rb 5 6 29.167k
eoa 5 0 4 6 100G
.ac lin 500 59 61
.probe
.end
Problem 6.55: Notch with OA with ft=1MHz
Vi 0 1 ac 1
r1 1 2 26.526k
r2 2 4 26.526k
r3 3 0 13.263k
c1 1 3 100n
c2 3 4 100n
c3 2 5 200n
ra 6 0 10k
rb 5 6 29.167k
eoa 5 0 Laplace {V(4,6)}={1Meg/(1+s/6.283)}
.ac lin 500 59 61
.probe
.end
```

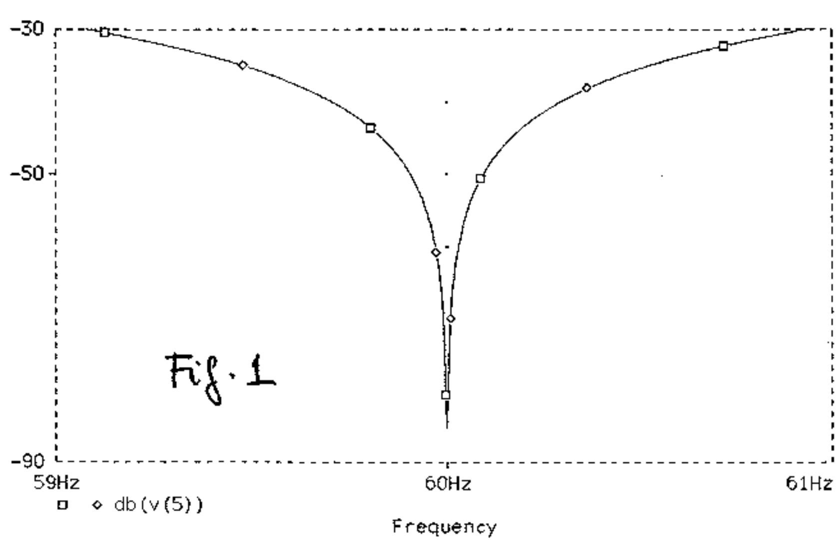
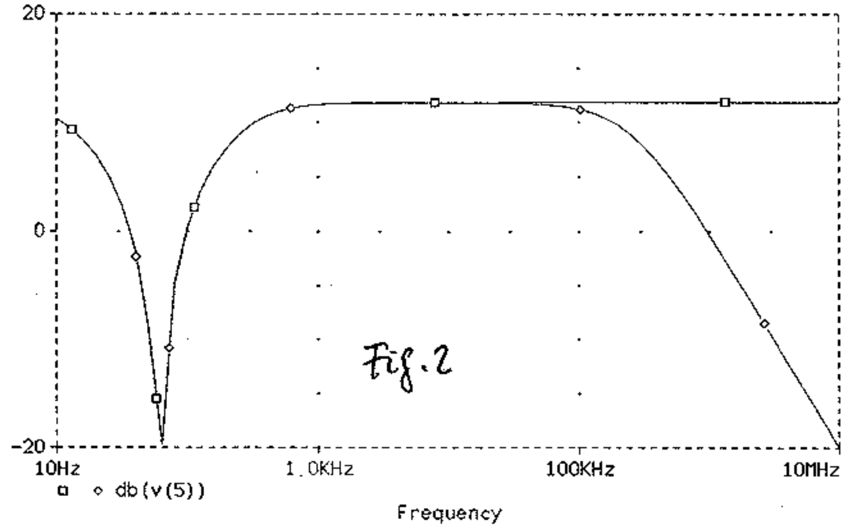
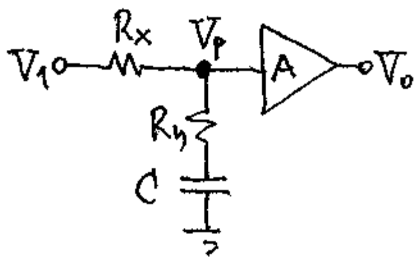


Fig. 1

6.40



6.56 (a)



$$V_o = AV_p = A\frac{R_y + 1/sC}{R_x + R_y + 1/sC}V_1$$

$$= \frac{A_0}{1+s/\omega_B}\,\frac{1+sR_yC}{1+s(R_x+R_y)C}V_1$$

Imposing $R_yC = 1/\omega_A$ and $R_x + R_y = R$ gives $V_o = A_0V_1/(1+sRC)$, i.e. the same relationship as an R-C stage followed by an ideal amplifier with gain A_0.

(b) $A_0 = 1\ \mathrm{V/V}$, $\omega_B = \omega_t = 2\pi f_t = 10^6\ \mathrm{Hz}$, $R_c = 1/2\pi f_t C = 159\ \Omega$ (use $158\ \Omega$, 1%), $R - R_c = 2199 - 159 = 2.04\ \mathrm{k\Omega}$ (use $2.05\ \mathrm{k\Omega}$, 1%).

(6.41)

6.57 $\left(\frac{1}{\beta}\right)_{min} = 10^3\ V/A$; $f_t = \frac{1}{2\pi\times10^3\times1.59\times10^{-12}} = 100\ MHz.$

Design for $T_0 = 0.5\times\frac{10^6}{10^3} = 500$ and $f_B = f_t$ in each case.

(a) $R_2/R_1 = 2$; $1+R_2/R_1 = 3$; impose

$R_2 + r_n(1+R_2/R_1) = 10^3 \Rightarrow R_2 = 10^3 - 25\times3 = 925\ \Omega$

(use $931\ \Omega$, 1%), $R_1 = \frac{1}{2}R_2 = 464\ \Omega$, 1%.

(b) $1+R_2/R_1 = 11$; $R_2 = 10^3 - 25\times11 = 732\ \Omega$, 1%, $R_1 = 73.2\ \Omega$, 1%.

(c) $R_1 = R_2 = 10^3 - 25\times2 = 953\ \Omega$, 1%.

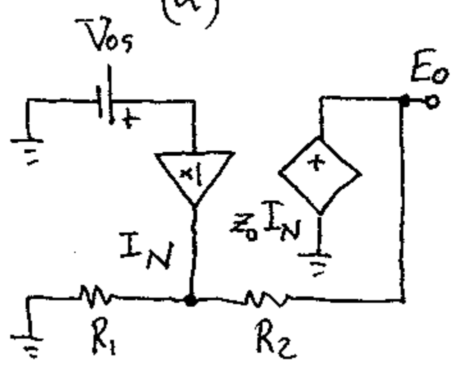
6.58 (a) $R_1 = \infty$, $R_2 = 10^3 - 25 = 975\ \Omega$

(b) $R_1 = R_2 = 10^3 - 25(1+1) = 950\ \Omega$

(c) $R_1 = \infty$, $R_2 = 2\times10^3 - 25 = 1975\ \Omega$

$R_1 = R_2 = 2\times10^{-3} - 50 = 1950\ \Omega$

(d)



$E_O = (1+R_2/R_1)V_{OS} + R_2 I_N$

(a) $E_{O(max)} = 1\times V_{OS} + 975\times I_N = 2.95\ mV$;

(b) $E_{O(max)} = 2V_{OS} + 950\ I_N = 3.9\ mV$.

(c) $E_{O(max)} = 4.95\ mV, 5.9\ mV$.

6.59 (a)

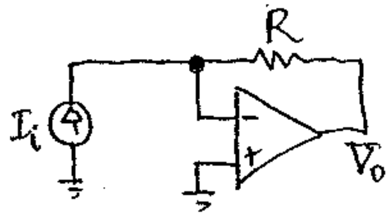


Fig. 1

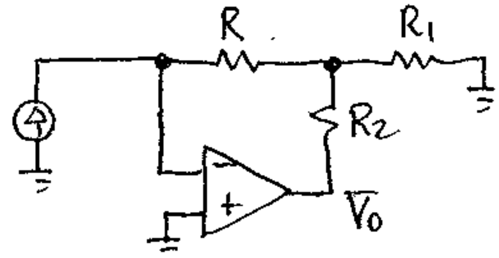


Fig. 2

$V_o/I_i = -10$ V/mA $= -10^4$ V/A $\Rightarrow R = 10$ kΩ in Fig. 1, and $R(1 + R_2/R_1 + R_2/R) = 10^4$ in Fig. 2. Pick $R = R_2 = 1$ kΩ; then $R_1 = 125$ Ω.

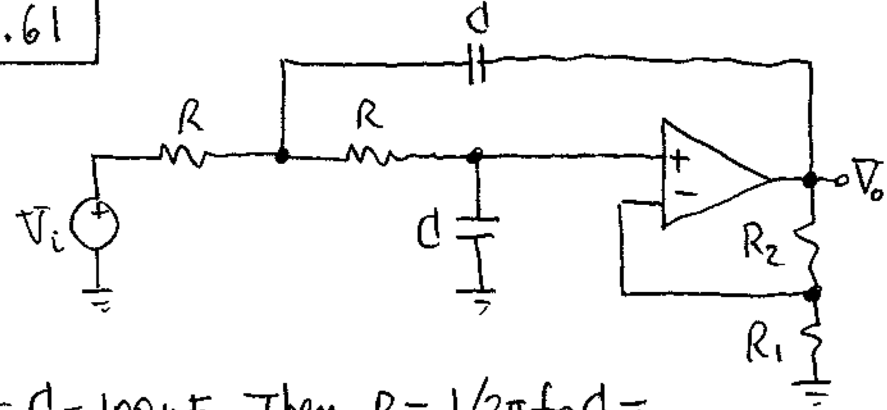
(b) In Fig. 1, $\beta = 1/(R + r_n) = 1/10{,}025$ A/V. $f_B = (10{,}000/10{,}025)f_t = 99.75$ MHz. $E_o = V_{OS} + RI_N$ $= 1$ mV $+ 20$ mV $= 21$ mV maximum.

In Fig. 2, $\beta = \dfrac{R_1}{R + r_n + R_1} \times \dfrac{1}{R_2 + R_1 \| (R + r_n)}$ $= 1/10{,}235$ A/V; $f_B = (10{,}000/10{,}235)f_t = 97.7$ MHz. $E_o = (1 + R_2/R_1)V_{OS} + 10^4 I_N = 8$ mV $+ 20$ mV $= 28$ mV maximum. Bandwidth is about the same; maximum error is worst in Fig. 2 because of the increased noise gain for V_{OS}.

6.60 Replace r_n with $r_n + R_{pot}$. Wiper at the right: $f_B = f_t/[1 + r_n/(1000 \| 110)]$ $= 10^8/[1 + 25/99.1] = 79.9$ MHz; $\tau_R \cong 2.2/(2\pi \times 79.9 \times 10^6) \cong 4.4$ ns. Wiper at the left: $f_B = 10^8/[1 + 1025/99.1] \cong 8.8$ MHz, $\tau_R \cong 39.7$ ns.

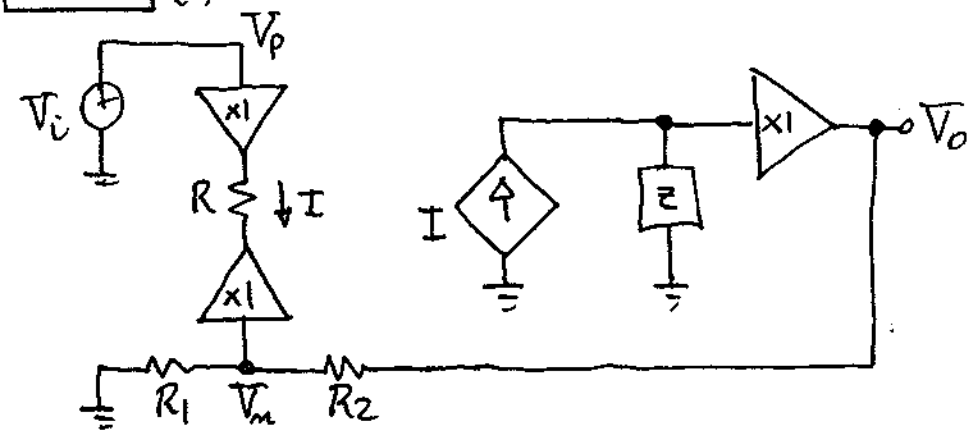
(6.43)

6.61



Let $C = 100$ pF. Then, $R = 1/2\pi f_0 C =$
$1/(2\pi 10^7 \times 10^{-10}) = 159\,\Omega$ (Use $158\,\Omega$, 1%).
$Q = 1/(3-K) = 5 \Rightarrow K = 1 + R_2/R_1 = 2.8 \Rightarrow R_1 =$
$R_2/1.8 = 1.5\times10^3/1.8 = 833\,\Omega$ (Use $845\,\Omega$, 1%)

6.62 (a)



$V_o = zI = z\dfrac{V_p - V_n}{R + 2r_o} = a(V_p - V_n)$, $a = \dfrac{z}{R+2r_o}$

$SR = I/C_{eq} = (V_P - V_N)/[(R+2r_o)C_{eq}]$.

(b) $a_0 = \dfrac{R_{eq}}{R+2r_o} = \dfrac{10^6}{500+2\times25} = 1818$ V/V

$f_b = 1/(2\pi R_{eq}C_{eq}) \cong 80$ kHz; $f_t = a_0 f_b = 145$ MHz.
$\beta = 0.5$; $T_0 = 909$; $A_0 = 2\times\dfrac{1}{1+1/909} = 1.9978$ V/V.
$f_B = 72$ MHz.

(c) $SR = \dfrac{1}{550\times2\times10^{-12}} = 909$ V/μs.